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Risk-Based Premiums for Insurance Guaranty Funds

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ABSTRACT

Insurance guaranty funds have been adopted in all states to compensate policyholders for losses resulting from insurance company insolvencies. The guaranty funds charge flat premium rates, usually a percentage of premiums. Flat premiums can induce insurers to adopt high-risk strategies, a problem that can be avoided through the use of risk-based premiums. This article develops risk-based premium formulas for three cases: a) an ongoing insurer with stochastic assets and liabilities, b) an ongoing insurer also subject to jumps in liabilities (catastrophes), and c) a policy cohort, where claims eventually run off to zero. Premium estimates are provided and compared with actual guaranty fund assessment rates.

PROPERTY-LIABILITY INSURANCE COMPANIES are levered firms in which debt capital is supplied by policyholders in the form of premiums paid at contract inception (Quirin and Waters [30]). In return, the insurer promises to pay claims, contingent upon the occurrence of specified events during a coverage period. Premiums are invested in marketable securities between the premium-payment and loss-payment dates. In many lines of insurance, claim payments cover a lengthy period of time extending far beyond the coverage period.

Incentive conflicts arise because insurers can gain by increasing asset risk following the policy issue date. This increases the value of the owners' call option on the residual value of the firm at the expense of commitments to policyholders. To some extent, policyholders foresee this and avoid companies that are likely to assume unstable risk-taking postures. A network of insurance brokers and financial rating firms has arisen to supply monitoring services, and a significant share of insurance costs is devoted to this function. Information generated by this system coupled with repeat buying (periodic negotiation of contracts) serves to protect policyholder interests. Reinsurance companies exist to monitor the behavior of managers and limit the risk of ruin, thus helping to prevent dilution of the value of policyholder claims.

In spite of the substantial resources devoted to private monitoring, government also engages in extensive solvency regulation. One rationale for government regulation is that private monitoring is less effective for individual buyers and small businesses than for large corporate buyers due to economies of scale in information acquisition. In addition, the consequences of insolvencies are more severe for individuals and small firms than for large firms since the latter are traded in capital markets, which provide an efficient mechanism for pooling

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diversifiable risk. Thus, like banks, insurers have both risk-averse and risk-neutral customers with different needs and resources. (See Kanatas [17] for a discussion of the banking case.)

Another rationale for solvency regulation is the existence of incentive conflicts between buyers of insurance and third-party claimants. Many important types of insurance (e.g., liability insurance) are purchased to satisfy legal obligations toward potential claimants who have no role in the contracting process. To the extent that buyers are "judgment-proof" or have limited liability, an incentive exists to purchase the least expensive (i.e., highest risk) insurance permitted by law. This is obviously detrimental to the interests of third-party claimants, who will be unable to collect if the insurer becomes insolvent. Thus, solvency regulation also is motivated by the contention that private markets fail to incorporate the full social costs of insurer insolvencies.[1]

State regulators monitor the financial condition of insurance companies through an extensive auditing system. Insurer financial statements are subjected to computerized audits each year by the National Association of Insurance Commissioners (NAIC). Companies failing four or more of eleven audit ratio tests are singled out for special regulatory scrutiny. Site audits are carried out by a team of state examiners every three to five years (Troxel and Breslin [33]). Other solvency regulations include investment portfolio restrictions, minimum capitalization requirements, and (in many states) rate regulation.[2]

A recent addition to the regulatory arsenal is the insurance guaranty fund, which reimburses policyholders and third-party claimants of insolvent insurers. Property-liability insolvency funds exist in all United States jurisdictions. Nearly all funds operate on a post-assessment basis: solvent companies are assessed an amount equal to the shortfall in assets of the insolvent firm. Assessments are a flat percentage of premium volume for all solvent companies (Duncan [12]).

The guaranty fund system differs from bank deposit insurance in that the ultimate guarantor is the private insurance industry rather than the government. This is possible because property-liability insurers, unlike banks, do not have "instantaneously putable" debt and hence are not susceptible to runs. The establishment of guaranty funds reflects a political judgment that the costs of insurer insolvencies should be spread throughout the insurance system rather than borne by specific policyholders and claimants. It also reflects the view that the private market has proven inadequate as a risk-sharing mechanism for certain types of policyholders, as mentioned above.

Questions have been raised about the efficiency of governmental solvency regulation. For example, Munch and Smallwood [26] found that minimum capital and surplus requirements reduce the number of insolvencies by restricting entry into insurance markets but do not reduce the insolvency rate. Other regulatory

[1] It should be noted that reinsurance does not solve this problem. Insurers adopting high-risk strategies either will purchase no reinsurance or will buy from high-risk reinsurance companies to satisfy regulatory requirements. Since high-risk strategies often will benefit stockholders, this problem is unlikely to be alleviated by resolving manager-owner incentive conflicts.

[2] Rate regulatory laws usually incorporate the requirement that rates not be "excessive, inadequate, or unfairly discriminatory". In most recent periods, rate regulation has tended to set maximum rather than minimum rates. See Harrington [14].

measures were found to have little or no effect on the number of insolvencies. It is argued below that guaranty funds in their present form have done little to improve this situation.

In adopting guaranty funds, little attention was paid to the impact the funds might have on firm incentives, regulatory costs, or the risk-return characteristics of insurance markets.[3] Inadequate consideration has been given to the use of private alternatives or less intrusive regulatory approaches to protect risk-averse policyholders. For example, the results of regulatory solvency tests would assist smaller policyholders in overcoming their information disadvantage and avoiding insurers with a high probability of failure. However, the release of this information has been ruled out by the NAIC. The incentive conflict between insurance buyers and third-party claimants would be more difficult to handle in a private market context. Properly designed guaranty funds would seem to be advantageous in dealing with this problem in that they provide the broadest possible sharing of risk and low transactions costs.

This article is motivated by a defect in the design of guaranty funds—the use of flat rather than risk-based premiums. The objective is to point out the consequences of using flat premiums and to develop techniques for calculating risk-based premiums. The premium formulas should also be useful in providing information on insurer financial stability and in pricing private-market insurance contracts.

Guaranty funds with flat premiums create adverse incentives in insurance markets. In a competitive market with perfect information and no regulation, the cost of an insurer's debt capital (the underwriting loss (profit)) would vary directly with the risk of the firm (i.e., the uncertainty regarding the firm's ability to pay policyholder claims). Hence, more risky firms would have to charge lower premiums in order to attract policyholders.[4]

In a regime with guaranty funds, the market penalty is replaced by the guaranty fund premium.[5] If the premium does not reflect insurer risk, the introduction of guaranty funds will lead to a gain in the market value of equity for more risky firms, and an incentive is created for firms to adopt more risky strategies. Regulatory costs may arise as more intensive government monitoring is required to verify risk levels. The number of insolvencies also may increase. The higher regulatory costs and any deadweight losses arising from insurer insolvencies will be spread throughout the insurance system, raising the average cost of insurance.

[3] For an analysis of some of these issues in banking, see Buser, Chen, and Kane [6] and Chan and Mak [8].

[4] It is assumed here that private monitoring provides buyers with accurate information on firm risk and eliminates any problems of adverse firm behavior following policy issue. The important point is that buyers will be indifferent between high- and low-risk insurers following the introduction of the guaranty fund. Hence, even if some high-risk insurers were exploiting information imperfections prior to the adoption of the fund, the "lemons" problem would still exist after the fund were established. It is implicitly assumed here that rate and investment regulation are not binding.

[5] Insolvency funds may not totally eliminate the costs of insolvencies from the policyholders' perspective. For example, the payment of claims may be delayed if the insurer becomes insolvent and the liabilities are transferred to the guarantor. In addition, guaranty funds usually impose a maximum payment per claim. See Duncan [12].

Risk-based premiums thus have several advantages. They reduce the potential for distortions in insurance company asset and liability portfolios that raise the probability of insolvency. They provide a potentially more effective and less expensive means of monitoring solvency, monitoring competition (determining fair rate of return), and providing information to policyholders in both regulated and unregulated insurance markets. Finally, the premium formulas have potential intrinsic value in pricing insurance and reinsurance contracts (e.g., loss portfolio transfers among insurers).

The methodology developed in this paper is an extension of work on deposit insurance premiums by Merton [24 and 25], Pennacchi [29], and Ronn and Verma [31]. The approach is to model the assets and liabilities of the firm as diffusion processes.[6] Premium equations are developed for three cases: a) an insurer with stochastic assets and liabilities but no additional sources of risk, b) an insurer with stochastic assets, stochastic liabilities, and randomly occurring jumps in liabilities (catastrophes), and c) a policy cohort, where liabilities are gradually reduced as claims are paid.

The first two models provide risk-based guaranty fund premiums for ongoing insurers, while the third provides the fair market value of the guarantor's promise to discharge the obligations of a bankrupt insurer. The first model extends the work of Merton [24 and 25] and Pennacchi [29] by respecifying the drift terms and boundary conditions of the relevant diffusions to apply to insurers rather than banks. The second model breaks new ground by allowing for discrete jumps in liabilities (catastrophes), a major problem faced by insurers. It differs from previous jump-diffusion models (e.g., Merton [23]) in that both assets and liabilities are stochastic and that the jumps are in liabilities (i.e., the denominator of the asset/liability ratio). The cohort model is totally new and has significant potential for pricing insurance contracts, loss portfolio transfers, and guaranty fund premiums.

Related work in insurance pricing includes Kraus and Ross [19] and Doherty and Garven [11].[7] The former utilize arbitrage pricing theory to develop equilibrium premiums for property-liability insurers, while the latter present a more conventional Black-Scholes model of the property-liability firm. The present model is a useful alternative to the Kraus-Ross model since it requires fewer parameter estimates and allows for catastrophes. It is superior to the Doherty-Garven approach since it explicitly allows for multiple-period operations, catastrophes, and the pricing of policy cohorts. The present model also makes allowance for the long payout tail that characterizes many important lines of insurance (e.g., liability insurance).

[6] An alterntive approach based on stable processes rather than diffusion processes is suggested by McCulloch [21]. Another potential methodology is actuarial-ruin theory (Beard, Pentikainen, and Pesonen [3]). Ruin theory, though mathematically sophisticated, does not take into account the operation of market forces.

[7] A recent work that applies modern financial theory to the pricing of *individual* insurance contracts is Sabol [32]. Earlier papers on the financial theory of insurance pricing include Fairley [13] and Hill and Modigliani [15].

The final contribution of the article is to provide numerical illustrations of risk-based premiums. These are not intended as a hypothesis test but rather to provide examples of potential estimation techniques and to indicate the order of magnitude of premiums generated by the models. The illustrative premiums are compared with actual guaranty fund assessment rates for the period 1970 to 1984.

I. Premiums for Ongoing Insurers

A. Stochastic Assets and Liabilities

Assume that the insurer enters into a contractual arrangement with the guaranty fund at the beginning of a specified contract period. The contract period is of fixed length (e.g., one year). The guaranty fund premium is determined and a premium charge made at the beginning of the period. At the end of the period, an audit occurs. If assets (A) exceed liabilities (L) at the audit date, the company is permitted to continue operating. A new premium is calculated, applying to the next contract period. If liabilities exceed assets, the guaranty fund takes over the assets of the company and discharges its liabilities at a cost to the fund of $L - A$. The fund is assumed to be certain to pay its obligations.

Except for the pre-payment of the guaranty fund premium, these assumptions are consistent with the current regulatory practices in insurance. Audits are nonrandom and conducted annually, with more extensive audits conducted at three- to five-year intervals. Audit costs are charged to insurers and can be considered proportional to the guaranty fund premium.

The following additional assumptions are made:

(A1) Trading in securities takes place continuously in time. Borrowing and lending take place at a known rate of interest r, which is assumed to be constant over time. The Fisher effect is assumed to prevail, i.e., $r = r_I + r^*$, where r_I is the economy-wide inflation rate and r^* is the real rate.

(A2) Security prices are assumed to satisfy the security market line equation in the continuous-time version of the capital asset pricing model (ICAPM).

(A3) Insurer assets consist of marketable securities. The value of these assets is determined according to the following diffusion process:

$$dA = (\mu_A A + \delta N - \theta L)dt + A\sigma_A dz_A, \tag{1}$$

where A = assets, L = liabilities, N = the number of policies, μ_A = the instantaneous expected rate of return on assets per unit of time, δ = the instantaneous rate of premium inflow per policy insured, θ = the instantaneous rate of claims payment per dollar of liabilities, σ_A^2 = the instantaneous variance of return on assets, and $z_A(t)$ = a standard Brownian motion process for assets. Assets thus are assumed to grow due to investment income and premium payments and to be reduced as claims are paid. Note that the model applies to the entire company rather than to any given entry-year class of policies.

(A4) Premiums are collected in advance and held until claims are paid. Liabilities are determined according to the following diffusion process:[8]

$$dL = (\mu_L L + \eta N - \theta L)dt + L\sigma_L dz_L, \qquad (2)$$

where μ_L = the instantaneous growth rate of liabilities, η = the instantaneous (dollar) rate of occurrence of new claims, σ_L^2 = the instantaneous variance of liabilities per unit of time, and $z_L(t)$ = a standard Brownian motion process for liabilities. Liabilities are assumed to grow due to inflation and the occurrence of new claims and to be drawn down due to the payment of claims.[9]

(A5) The asset and liability processes are related as follows:

$$dz_A dz_L = \rho_{AL} dt, \qquad (3)$$

where ρ_{AL} can be interpreted as an instantaneous correlation coefficient.[10]

The model assumes that both general inflation and liability inflation are constant over time. However, liability inflation will not necessarily equal general inflation. If liabilities have systematic risk, under assumption (A2) insurance prices will reflect a CAPM-type market risk premium.[11] Hence,

$$\mu_L = r_L + \pi, \qquad (4)$$

where r_L = the inflation rate in liabilities, which may be $\gtreqqless r_I$, and π = the market risk premium for bearing insurance risk, where π may be $\gtrless 0$. This equation implies that insurers promise to pay losses at the price level in effect at the loss-settlement date. However, deducted from outstanding liabilities is a risk charge, which constitutes the insurer's return for bearing insurance risk. This is similar in concept to the risk-adjusted discount rate used by Myers and Cohn [27] in a discrete-time context.

The value of guaranty fund insurance can be written as $P(A, L, \tau)$, where τ = the time remaining until the next audit. The guaranty is similar to a put option since its value at the audit date is $\max(0, L - A)$. Assets and liabilities both drift stochastically, and the guaranty fund assumes the insurer's obligations if liabilities have drifted above assets by the audit date.

Utilizing procedures analogous to those employed in Merton [25], a differential

[8] If N can be expressed as a constant proportion of liabilities, equation (2) asserts that insurer liabilities follow a lognormal diffusion process. Cummins and Nye [10] have provided evidence that the lognormal distribution is a reasonable model for insurer liabilities.

[9] Conceptually, η is designed as a diffusion-type approximation of the total claims process. The latter process is explained in Beard, Pentikainen, and Pesonen [3]. It involves the compounding of frequency (claims occurrence) and severity (claim amount) processes. Thus, η is dollar valued and represents the infinitesimal growth in liabilities due to new claims.

[10] Much empirical work suggests that asset and liability processes of insurers have very low or no correlation (e.g., Hill and Modigliani [15] and Cummins and Harrington [9]). However, for completeness, the models are developed under the assumption that nonzero corrrelations may be present.

[11] Discrete-time insurance-pricing models with this type of risk premium have been developed by Fairley [13] and Hill and Modigliani [15].

equation for the guaranty fund premium is obtained:[12]

$$rP = (rA + \delta N - \theta L)P_A + (r_L L + \eta N - \theta L)P_L - P_\tau$$
$$+ \tfrac{1}{2}(A^2\sigma_A^2 P_{AA} + L^2\sigma_L^2 P_{LL} + 2AL\sigma_A\sigma_L\rho_{AL}P_{AL}). \quad (5)$$

This equation can be solved numerically to obtain the guaranty fund premium.

To facilitate the discussion and elucidate the key relationships, equation (5) is simplified by making an additional assumption: $\delta N = \theta L = \eta N$. That is, the insurer is assumed to have attained a *steady-state* position where premium inflow, claims outflow, and the (dollar) incidence of new claims are equal. These assumptions are merely an expositional convenience and are not necessary to obtain premium rates.

Making the change of variables $x = A/L$ and $p(x) = p = P(A, L, \tau)/L$ in equation (5), the following equation is obtained:

$$(r - r_L)p = (r - r_L)xp_x - p_\tau + (\tfrac{1}{2})x^2 p_{xx}(\sigma_A^2 + \sigma_L^2 - 2\sigma_A\sigma_L\rho_{AL}). \quad (6)$$

The boundary conditions for equation (6) are the following:

$$p(0, \tau) = \exp[-(r - r_L)\tau], \quad (7a)$$

$$p(x, 0) = \max(0, 1 - x). \quad (7b)$$

The guaranty fund premium thus is the value of a put option with interest rate $(r - r_L)$. If claims inflation is the same as general inflation, this is equivalent to valuing the option at the real rate of interest, r^*. The diffusion parameter is $(\sigma_A^2 + \sigma_L^2 - 2\sigma_A\sigma_L\rho_{AL})$. Thus, positive correlation between the asset and liability processes reduces the risk of the insurer.

The model can be used to analyze the impact of guaranty fund insurance on firm value.[13] Insurance liabilities can be considered analogous to risky corporate debt; i.e., the value of liabilities will be

$$V(L) = L[\exp(-r^*\tau) - p(x, \tau)], \quad (8)$$

where $V(L)$ = value of liabilities. Liabilities will be worth less than their discounted present value due to the probability that the insurer will default. The value of insurer assets is

$$A = E + L[\exp(-r^*\tau) - p(x, \tau)], \quad (9)$$

where E = insurance company equity. Recalling that insurer assets are marketable securities, it is clear from equation (9) that the assets are obtained from two sources: stockholder funds (E) and premiums (discounted claims less the value of the put).

There is, of course, an interaction between the funds initially put up by

[12] The present model differs from Merton's [25] model in that both assets and liabilities are stochastic. The derivation utilizes Itô's Lemma and assumes that the deposit insurance premium is priced according to the continuous-time version of the CAPM. The details of the derivation are available from the author.

[13] This discussion assumes for simplicity that insurer liabilities have no systematic risk.

stockholders and the value of the put; i.e., the put value is affected by firm capital structure. It is assumed that insurance market equilibrium results in insurers being arrayed along a continuum of capital structures to serve policyholders with different levels of risk aversion. Policyholders are assumed to have complete information about the default risk of competing insurers and to choose a company with a premium/default risk combination consistent with their tastes.

Suppose that guaranty fund insurance is introduced with a flat premium k expressed as a proportion of liabilities. Suppose that all existing policies are cancelled and replaced by new policies reflecting the existence of the guaranty fund. All insurers will now receive a premium of $\exp(-r^*\tau)$ per dollar of liabilities from policyholders and pay the guaranty fund a premium of kL. The value of the firm's assets becomes

$$A' = E + L[\exp(-r^*\tau) - k]. \quad (10)$$

The change in the asset value of any given insurer due to the introduction of the guaranty fund will be

$$A' - A = L[p(x, \tau) - k]. \quad (11)$$

The flat premium charge k will be a pooled value based upon the insolvency costs of all insurers in the market. Thus, k will be higher than $p(x, \tau)$ for some insurers and lower for others. The introduction of the guaranty fund will penalize the equityholders of better-than-average firms and reward equityholders of worse-than-average firms.

Viewing equity as the value of a call option, it is apparent that the firm has an incentive to assume greater risk.[14] This is the case because the value of the call option increases with the risk parameter $(\sigma_A^2 + \sigma_L^2 - 2\sigma_A\sigma_L\rho_{AL})$. Insurers are no longer penalized by the product market for assuming greater risk and are penalized by the guaranty fund only as a group (through increases in k as risk increases). Hence, the introduction of guaranty funds may actually increase the rate of insolvency in insurance markets. Charging a risk-based premium reinstates equation (9), with insurers paying a premium of $Lp(x, \tau)$ to the guaranty fund.[15]

B. Liability Jumps (Catastrophes)

The lognormal diffusion process hypothesized above implies a continuous sample path for liabilities. However, many types of insurance subject the insurer to large jumps in liabilities (catastrophes). Examples include chemical catastrophes, airline disasters, hurricanes, and changing judicial interpretations of contract provisions. Although reinsurance is used to cushion the impact of such events, it is impossible to perfectly insulate insurers from this type of claim. This

[14] The value of insurance company equity will be equal to the value of a call option, valued according to a differential equation exactly analogous to equation (6), with boundary conditions: $c(0, \tau) = 0$, $c(x, 0) = \max(0, x - 1)$.

[15] However, policyholders will pay higher premiums, reflecting the reduction in default probabilities. This may reduce overall consumer welfare because policyholders who prefer to take some default risk will not have this option available in insurance markets.

section develops a pricing model for guaranty fund insurance when insurers are subject to discontinuous changes in liabilities.

Assets are assumed to be governed by equation (1), while liabilities now develop according to the following equation:

$$dL = (\mu_L L + \eta N - \theta L - \lambda k L)dt + \sigma_L L dz_L + dq, \tag{12}$$

where $q(t)$ = a Poisson process with parameter λ, $k = \mathrm{E}(Y - 1)$, and Y = the random variable representing the jump magnitude, where $Y > 0$. Jumps occur according to a Poisson process; i.e., the intervals between jumps are exponentially distributed.[16] When a jump occurs, liabilities change from Y to LY. Successive values of Y are assumed to be independent and identically distributed. The expected impact of the jump process in any small interval is λkdt. A similar model has been applied to stock prices by Merton [23] and other authors. The difference between the present model and Merton's [23] model is that both assets and liabilities are stochastic here and the jump applies to liabilities rather than assets (i.e., it is in the denominator of the asset/liability ratio).

Representing the guaranty fund premium by $P(A, L, \tau)$ and applying the Itô transformation formula and a corresponding formula for Poisson processes (see Merton [22, pp. 395–396]), a differential equation for the premium $P(A, L, \tau)$ can be obtained.[17] Two assumptions are invoked in order to eliminate the Brownian motion terms from the differential equation: a) the guaranty fund premium is priced according to the continuous-time version of the CAPM, and b) jump risk is nonsystematic. The first assumption is retained from the preceding case, while the second is needed because no hedge portfolio can be constructed that will eliminate the jump risk. (See Merton [23, p. 131].)

Assuming that the jump risk is nonsystematic is probably more reasonable for insurance liabilities than for stock prices. While certain "catastrophes" (e.g., liberalization of tort rules) undoubtedly have a systematic effect, most such events are unsystematic by definition. Nevertheless, to the extent that jump risk has a systematic component, the guaranty fund premium may be understated.

Introducing the above assumptions and taking expectations yield a differential equation for the guaranty fund premium. After making the change of variables $x = A/L$ and $p(x, \tau) = P/L$, the equation becomes

$$pr^{**} = xp_x r^{**} - p_\tau + \lambda \mathrm{E}_Y[p(x/Y, \tau) - p(x, \tau)]$$
$$+ (1/2)x^2 p_{xx}(\sigma_A^2 + \sigma_L^2 - 2\sigma_A \sigma_L \rho_{AL}), \tag{13}$$

where $r^{**} = r - r_L + \lambda k$. The solution to equation (13) is given below:

$$p(x, \tau) = \textstyle\sum_{N=0}^{\infty} p(N)\mathrm{E}_N[W(x/X_N, \tau; 1, r^{**}, \sigma^2)], \tag{14}$$

where X_N = the product of N independent random variables, each distributed identically to Y, E_N = the expectation operator over the distribution of X_N,

[16] The Poisson model is the premier model of insurance claim arrivals. See Beard, Pentikainen, and Pesonen [3].

[17] The derivation is somewhat analogous to that presented in Merton [23]. However, since both assets and liabilities are stochastic and the jumps apply to liabilities, the analogy is not direct. Details of the derivation are presented in an Appendix available from the author.

$W(x/X_N, \tau; 1, r^{**}, \sigma^2)$ = the value of a put option on an asset with value x/X_N, time to expiration τ, exercise price one, interest rate $r - r_L + \lambda k$, and risk parameter σ^2, and $\sigma^2 = \sigma_A^2 + \sigma_L^2 - 2\sigma_A\sigma_L\rho_{AL}$. Equation (14) is the weighted average of guaranty fund premiums with asset/liability ratios x/X_N and exercise price one. In effect, the occurrence of catastrophes leads to multiplicative increases in liabilities and hence to decreases in the asset/liability ratio.

No distributional assumption has been made with regard to the random variable Y. However, if Y is lognormal, then a closed-form expression can be derived for the value of the guaranty fund premium. This is equation (14) with the following substituted for $E_N[W(\cdot)]$:

$$E_N[W(\cdot)] = e^{-r^{**}\tau}\Phi[(-\ln x - \mu^*\tau)/\sigma^*\sqrt{\tau}] - xe^{N\gamma}\Phi[(-\ln x - \mu^*\tau - \sigma^{*2}\tau)/\sigma^*\sqrt{\tau}], \tag{15}$$

where $\mu^* = r - r_L + \lambda k - \sigma^2/2 - N\alpha/\tau$, $\sigma^{*2} = \sigma^2 + (N\zeta^2/\tau)$, $\Phi(\cdot)$ = the standard normal distribution function, α, ζ^2 = the mean and variance parameters of $\ln(Y)$, and $\gamma = -\alpha + \zeta^2/2$. The derivation of this equation utilizes the property that the product of lognormally distributed random variables is lognormally distributed.[18] The expected value of the jump, $E(Y)$ is $\exp(\alpha + \zeta^2/2)$. The expected impact of the jump on the asset/liability ratio is $E(1/Y) = \exp(-\alpha + \zeta^2/2)$.

If the Poisson intensity parameter and the liability-jump parameters are equal to zero, the jump model reverts to the no-jump case. However, if the expected value of the jump is zero (i.e., $\alpha = -\zeta^2/2$) but jumps can occur ($\lambda \neq 0$), the premium implied by equations (14) and (15) tends to be higher than in the no-jump case when $x > 1$. This occurs primarily because of the effect of the jump-variance parameter on σ^{*2}. When x is sufficiently less than one, the premium implied by the jump model when $E(Y) = 1$ tends to be less than the premium in the no-jump case. The reason is that the expected impact of the jump on x, i.e., $E(1/Y)$, is positive when $\alpha = -\zeta^2/2$.

II. Premiums for a Policy Cohort

In many cases of insurance company financial distress, the liabilities of the distressed firm are not actually discharged by the guaranty fund. Rather, the liabilities and assets are assumed by a solvent insurer that agrees to pay the remaining claims. This type of arrangement is rational if insurers have either cost or information advantages over the guaranty fund in claims settlement. In such situations, it is important to determine the fair market value of the business transferred to the solvent company. This section develops a pricing model for this type of transaction. The model also is appropriate for pricing the transfer of blocks of business between solvent firms, so-called "loss-portfolio transfers." A

[18] Equations (14) and (15) do not simplify into a weighted sum of Black-Scholes option values with Poisson parameter $\lambda' = \lambda(1 + k)$ as in Merton [23, p. 135]. This is due to the fact that $\ln(1 + k) = \alpha + \zeta^2/2$, while $\ln E(1/X_N) = -N\alpha + N\zeta^2/2$. The difference is that the jump affects the denominator rather than the numerator, as in Merton's jump model for stocks.

noteworthy feature of the model is that it prices a contingent claim with no expiration date and thus explicitly allows for a lengthy payout tail.

More specifically, consider a situation in which the insurer agrees to pay all claims arising from a group of policies in return for a fixed premium, G, to be paid at the inception of the contract. Assume that all claim events under the block of policies have already occurred and that the present market value of these claims (L) is known. The funds represented by G are invested in marketable securities. G and L are governed by the following differential equations:

$$dG = (\mu_A G - \theta L)dt + G\sigma_A dz_A, \tag{16a}$$

$$dL = (\mu_L L - \theta L)dt + L\sigma_L dz_L, \tag{16b}$$

where the parameters of the diffusions are defined above.

The loss account, represented by L, increases due to claims inflation and is drawn down as claims are paid. The premium account grows due to investment income and is depleted by claims payments. If the premium account eventually is exhausted, the insurer is still liable for any claims remaining unpaid. It is assumed that the insurer is certain to fulfill this obligation. The value of the insurer's obligation is denoted by $\Pi(G, L)$.

The value of $\Pi(G, L)$ is derived conditional on the initial values of G and L being known. For example, if the guaranty fund or a solvent insurer were to assume control of an insolvent insurer, both assets and liabilities would have known or estimable market values at the time of the takeover and would be subject to stochastic variation thereafter. This is precisely the situation modeled in this section.

Differentiating $\Pi(G, L)$, one obtains

$$d\Pi = \Pi_G dG + \Pi_L dL + \tfrac{1}{2}[\Pi_{GG}(dG)^2 + \Pi_{LL}(dL)^2 + 2\Pi_{GL}dGdL]. \tag{17}$$

As before, subscripts on Π indicate partial derivatives. To obtain a differential equation for π, the guaranty premium per dollar of liabilities, the following procedure is followed: a) equations (16a) and (16b) are substituted into (17), b) the expected value of the resulting expression is obtained under the assumption that the guaranty value reflects only systematic risk, and c) the change of variables $x = G/L$ and $\pi = \Pi/L$ is carried out. The result is the following equation:

$$(r - r_L + \theta)\pi = \pi_x[(r - r_L + \theta)x - \theta] + \tfrac{1}{2}x^2\pi_{xx}(\sigma_G^2 + \sigma_L^2 - 2\sigma_G\sigma_L\rho_{GL}). \tag{18}$$

The boundary conditions are

$$\lim_{x\to 0}\pi(x) = 1, \tag{19a}$$

$$\lim_{x\to\infty}\pi(x) = 0. \tag{19b}$$

The conditions imply that the value of the guaranty approaches zero as the asset/liability ratio (x) becomes large and approaches the value of liabilities as x goes to zero. Thus, if the premium account is exhausted, the value of the guarantor's promise is equal to the value of the remaining liabilities.

The solution of equation (18) subject to the boundary conditions is[19]

$$\pi(x) = \frac{\Gamma(2)}{\Gamma(2 + a)} b^a x^{-a} e^{-b/x} M(2, 2 + a, b/x), \qquad (20)$$

where $a = 2(r^* + \theta)/Q$, $b = 2\theta/Q$, $Q = \sigma_G^2 + \sigma_L^2 - 2\sigma_G\sigma_L\rho_{GL}$, and $M(\cdot, \cdot, \cdot) =$ Kummer's function. (See Abramowitz and Stegun [1].) The derivative of $\pi(x)$ with respect to x is unambiguously negative; i.e., the value of the guaranty is less when the premium account is larger relative to the loss account. For reasonable parameter values (see below), $\pi(x)$ varies inversely with the risk-free rate and directly with the risk parameter Q.

The effect of the payout parameter θ on the value of the guaranty is ambiguous. For large values of x (e.g., x sufficiently greater than one), $\pi(x)$ tends to be inversely related to θ. Intuitively, this occurs because a faster payout reduces the risk that adverse loss fluctuations will exhaust the premium fund. For sufficiently small values of x, on the other hand, the value of the guaranty tends to be directly related to the payout parameter. In this case, a proportion of the liabilities is very likely to be paid out of the guarantor's funds (rather than the premium account). Thus, it is best to delay payment as long as possible. Numerical examples are utilized in the next section to illustrate these and other effects.

III. Numerical Examples

This section provides numerical examples of guaranty fund premia. The examples are purely illustrative and do not constitute a test of hypotheses regarding the models developed above. The goal is to provide an illustration of possible estimation techniques and of the orders of magnitude of the premiums implied by the models. The premium estimates are compared with actual guaranty fund assessment rates for the period 1970 to 1984.

The most basic guaranty fund model for an ongoing firm depends upon only five parameters: the risk-free rate (r), the liability inflation rate (r_L), the variance parameters for assets and liabilities (σ_A^2 and σ_L^2), and the correlation parameter of the asset and liability random shock terms (ρ_{AL}).

Two estimates of the "real" rate of return $r^* = r - r_L$ are used, 0.005 and 0.025. The former is approximately the same order of magnitude as the realized difference between ninety-day U.S. Treasury bill rates and inflation over the period 1926 to 1984, while the latter is more in accord with realized rates over more recent holding periods (Ibbotson Associates [16]).

It is assumed that insurers invest twenty-five percent of their assets in corporate equities and seventy-five percent in long-term bonds—a reasonable approximation of insurer asset portfolios over the past ten years. This assumes that noninvested assets (about fifteen percent of insurer assets) have the same risk characteristics as long-term bonds. (See Hill and Modigliani [15].)

To estimate the diffusion parameters of corporate equities and long-term bonds, annual data presented in Ibbotson Associates [16] are used. Parameters

[19] The derivation of equations (18) through (20) is presented in an Appendix available from the author.

were estimated for stocks, long-term corporate bonds, and long-term government bonds for the period 1926 to 1984.[20] The average of the long-term corporate and long-term government bond parameters (0.00465) was used as the diffusion parameter for bonds. For equities, the estimated parameter was 0.0415. The correlation parameter between stocks and bonds is the average of the stock/government and stock/corporate bond parameters (0.115).

The risk parameter for insurance industry liabilities was estimated to be 0.0045, based on the variance of the log of (L/L_{-1}), where L = total industry liabilities. The correlation coefficient between liabilities and assets was assumed to be zero.[21]

To estimate premiums using the jump-diffusion (catastrophe) model, three additional parameters are needed: the Poisson arrival rate and the parameters of the lognormal distribution of catastrophe magnitudes. Annual arrival rates of 0.33, 0.2, and 0.1 were arbitrarily selected, corresponding to the average arrival of catastrophes every three, five, and ten years, respectively. For catastrophe severity, the location parameter was assumed to be −0.005 and the dispersion parameter 0.01. This implies that the expected impact of a catastrophe is zero; i.e., "catastrophes" can either increase or decrease insurer liabilities.[22] The assumed-dispersion parameter implies that the standard deviation of catastrophe severity is ten percent of insurer liabilities.

The final parameter estimate needed to test the premium models is the claims-settlement parameter (θ). The setup of the model implies that the claims runoff follows an exponential distribution.[23] Analysis of automobile bodily injury liability insurance claims from Massachusetts revealed that the exponential distribution provides a good fit to the tail of the claims runoff, with a parameter (θ) of 0.4. This estimate is less than the runoff parameter for the industry as a whole because insurers write both short-tail (e.g., fire) and long-tail (e.g., auto liability) lines.

The premium estimates for the basic and catastrophe models are shown in Table I. Premiums were estimated using asset/liability ratios ranging from 1.2 to 1.4.[24] The premium estimates for the basic model with a real rate of 0.005 range from 0.13 to 0.001 of one percent of liabilities for asset/liability (A/L) ratios of 1.2 and 1.4, respectively. For a real rate of 0.025, the premium is reduced

[20] The lognormal-diffusion assumption implies that the logs of value relatives are normally distributed. Hence, an approximate estimate of the diffusion parameter is the variance of the log of the value relative.

[21] The estimate of the liability variance is almost identical to the variance of $\ln(L/L-1)$ about its least-squares trend line. The estimated correlation parameters between $\ln(L/L_{-1})$ and the log asset relatives were very low (the largest was −0.16). Risk parameters for individual firms probably would be larger because a pooling effect occurs when aggregating across companies.

[22] The jump-severity parameters were chosen arbitrarily because data on catastrophe magnitudes are not readily available. The model can easily be modified to test the sensitivity of the estimated premiums to the jump-severity assumption and to reflect only non-negative jumps.

[23] Thus, the average claim-payoff date occurs $(1/\theta)$ periods after the starting date (time zero). The exponential-runoff assumption also was used by Kraus and Ross [19].

[24] The asset/liability ratio for the insurance industry over the past ten years has been about 1.4 (A. M. Best Company, *Best's Aggregates and Averages* (Oldwick, NJ)). However, this is probably overstated since bonds are reported at book rather than market values.

Table I

Guaranty Fund Premiums[a]

	Jump Model ($r^* = 0.005$)			No Jumps	
A/L Ratio	$\lambda = 0.33$	$\lambda = 0.2$	$\lambda = 0.1$	$r^* = 0.005$	$r^* = 0.025$
1.2	0.002789	0.002194	0.001741	0.001293	0.000753
1.3	0.000645	0.000430	0.000275	0.000131	0.000067
1.4	0.000159	0.000091	0.000047	0.000010	0.000004

[a] Constant parameters: $\alpha = -0.005$; $\zeta^2 = 0.01$. Key: α = location parameter for jumps; ζ^2 = dispersion parameter for jumps; r^* = the real rate (the nominal risk-free rate minus the liability inflation rate); λ = Poisson arrival rate for jumps; A = assets; L = liabilities.

Table II

Policy Cohort Premiums[a]

$Q =$	0.01	0.02	0.01	0.01	0.02
$r^* =$	0.005	0.005	0.025	0.005	0.005
$\theta =$	0.4	0.4	0.4	0.2	0.2
x	Premiums = $\pi(x)$				
1.4	0.000059	0.001272	0.000011	0.001011	0.007341
1.2	0.002317	0.009964	0.000708	0.008433	0.024921
1.0	0.038678	0.057026	0.020500	0.051495	0.077266
0.8	0.190896	0.195087	0.152593	0.186007	0.198244
min x	0.772	0.698	0.792	0.718	0.634
min $x + \pi$	0.991	0.992	0.953	0.983	0.985

[a] Key: Q = sum of asset and liability risk parameters; r^* = the real rate; θ = liability payout parameter; x = asset/liability ratio.

by about forty percent for A/L of 1.2 and by about sixty percent for A/L of 1.4. The risk parameter also has a major impact. For the no-jump case with $r^* = 0.005$ and $A/L = 1.2$, raising the variance parameter (σ^2) from 0.01 to 0.02 leads to a premium increase of more than five hundred percent. The premiums with catastrophes are uniformly higher.

To gauge the realism of the results, the actual guaranty fund assessment rate over the period 1970 to 1984 was computed.[25] The assessments for this period amounted to about 0.025 of one percent of industry liabilities. This is the same order of magnitude as many of the premium estimates in Table I. For example, it is about the same as the estimate for the no-jump model with $r^* = 0.005$ and $A/L = 1.275$.

The premium estimates for the policy-cohort model are shown in Table II. As expected, the premiums vary inversely with the asset/liability ratio and the real rate and directly with the variance parameter. As explained above, the effect of the runoff parameter (θ) depends upon the initial asset/liability ratio. For example, consider the second and fifth columns in Table II. For an asset/liability

[25] The first full year of operation of insurance guaranty funds was 1970. The assessment data (see National Committee on Insurance Guaranty Funds [28]) slightly understate the actual premium rate since the data base omits the pre-assessment funds operating in New York and in New Jersey prior to 1975.

ratio of 1.4, decreasing the runoff parameter (i.e., increasing the length of the payout period) increases the value of the guaranty. For A/L of 0.8, on the other hand, a longer payout period tail leads to a lower premium.

If $r^* > 0$, the sum of $A/L = x$ and the premium π has a unique minimum for any given set of parameter values.[26] The minima are shown in Table II for each parameter set tested. The accompanying minimum values of x also are shown. Increasing the risk parameter σ^2 reduces the value of x at which the minimum is attained but increases the sum of x and the guaranty premium. Lengthening the payout tail reduces the minima of both x and $x + \pi$.

Even though the present value of liabilities (per dollar of liabilities) is one, the minimum of $(x + \pi)$ is less than one. This result obtains as long as $r^* > 0$. If $r^* = 0$, $x + \pi$ approaches one from above.

IV. Summary and Conclusions

This paper develops premium calculation models for insurance guaranty funds. Three models are developed: a) a basic model for a one-year guaranty applicable to an ongoing company, b) a one-year premium model with catastrophes, again for an ongoing company, and c) a policy cohort model for a block of policies with no fixed expiration date.

The models are based on the assumption that the paths of assets and liabilities over time can be described by diffusion processes. They also assume that the value of the guaranty fund itself reflects only systematic risk. The guaranty fund is assumed to be certain to pay its obligations.

The premium models account for asset risk, liability risk, and the risk of catastrophes. Unlike most previous pricing models for insurance risk (e.g., Fairley [13]), the models developed in this paper recognize the value of the guarantor's promise to pay losses even if the premium account has been exhausted. Thus, they provide a link between financial-theory and traditional actuarial-ruin models.

Additional research is needed to provide parameter estimates that take into account the asset and liability portfolio characteristics of individual companies. Further analysis of the liability-runoff and claims-inflation processes also would be helpful. Potential theoretical extensions include the effects of stochastic interest and jump processes other than the Poisson. If this research can be conducted successfully, the models developed in this paper have the potential to yield significant improvements in the pricing techniques utilized in insurance.

Finally, further research should be undertaken on the overall impact of solvency regulation on insurance markets. The objectives of regulation should be specified more clearly, and the assumption that the regulator's goal is the continued solvency of all presently existing insurers (see, for example, Mayerson [20, p. 150]) should be critically examined. This research should consider the joint effects of regulations such as minimum capitalization requirements, price regulation, asset portfolio restrictions, and guaranty funds. The goal should be to

[26] The proof is available from the author.

define economically viable regulatory objectives and to specify the combinations of policy variables that can achieve these objectives most effectively. The advantages and disadvantages of using private market mechanisms to accomplish regulatory goals should be thoroughly explored. By providing market prices for risks traded in insurance markets, the models presented in this article should play a central role in future research on the general effects of solvency regulation.

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