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Dynamic risk management: Theory and evidence ☆

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Abstract

We present and test an infinite-horizon, continuous-time model of a firm that can dynamically adjust the use of risk management instruments which seek to reduce product price uncertainty and thereby mitigate financial distress losses and reduce taxes. The dynamic setting relaxes several restrictive assumptions common to static models. In the model, the firm can adjust its use and the hedge ratio and maturity of risk management instruments over time, risk management instruments expire as time progresses, the available maturity of the risk management instruments is shorter than the lifetime of the firm, and transaction costs are associated with initiation and adjustment of risk management contracts. The model produces a number of new time-series and cross-sectional implications on how firms use short-term instruments to hedge long-term cash flow uncertainty. Numerical results describe the optimal timing, adjustment, and rollover of risk management instruments and the choice of contract maturity and hedge ratio in response to changes in the firm's product price. The results show that the structure of transaction costs can have an important effect on the firm's risk management strategy. The model predicts that firms that are either far from financial distress

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or deep in financial distress neither initiate nor adjust their risk management instruments, while firms between the two extremes initiate and actively adjust their risk management instruments. Using quarterly panel data on gold mining firms between 1993 and 1999, we find evidence of a non-monotonic relation between measures of financial distress and risk management activity consistent with the model. We also provide evidence supportive of the model's predictions with respect to the maturity choice of risk management contracts.

JEL classification: G30; G32; G33

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1. Introduction

Much of the understanding of corporate risk management is based on static models that describe how various capital market imperfections give firms an incentive to reduce risk. While existing models provide rich intuition as to why firms should manage risk, they provide fewer predictions about how firms translate the incentives to manage risk into actual decisions on the choice of risk management instruments and how these strategies evolve over time.

Our main contribution is to present and test a dynamic model of corporate risk management in a continuous-time and infinite-horizon framework.[1] We analyze issues, which are difficult to address in static models, including the optimal timing to initiate risk management contracts, early termination, replacement of expiring and terminated contracts, the choice of maturity and hedge ratio, and frequency of adjustment. Many static models assume that firms make one-period decisions to hedge and that these decisions are irreversible and costless.[2] Therefore one-period models also often implicitly assume that the employed risk management instruments have the same duration as the lifetime of the firm. Treating risk management choices as irreversible restricts the ability of static models to recognize the value of dynamic risk management in adapting to changes in market conditions and firm characteristics. The fact that most risk management instruments have shorter maturities than the duration of the firm's operations has important implications for the timing and sequence of risk management decisions, and it provides an intuition for the limited effect of risk management on firm exposures observed in empirical studies such as Géczy et al. (1999), Petersen and Thiagarajan (2000), and Allayannis et al. (2003).

Following the static model of Smith and Stulz (1985), the model motivates risk management mainly via financial distress costs that are incurred when the firm's product price declines below costs and leverage exceeds a predetermined critical

[1]Other papers on hedging that use a continuous-time framework include Stulz (1984); Ho (1984) and Leland (1998).

[2]Throughout the remainder of the paper the terms hedging and risk management are used interchangeably.

level.[3] As a consequence, the model captures the suggestion by Stulz (1996) that firms use risk management not to reduce volatility per se but to avoid costly lower-tail outcomes that lead to financial distress or to reduce the length of time the firm spends in distress. We also provide results regarding tax code convexity as a motivation for risk management. In the model, the firm chooses the timing, maturity, and hedge ratio of risk management contracts in which the maximum available maturity is shorter than the duration of the firm's operations and expected cash flows. Risk management contracts are modeled as a portfolio of forward contracts (with an initial contract value of zero) on the firm's product price, which is the source of uncertainty in the model evolving as a stochastic process.

In the model, the firm faces the question of how to use short-term instruments to hedge long-term operations, given that both the initiation and early termination of risk management contracts incur transaction costs.[4] Moreover, in a multi-period dynamic setting, the firm constantly faces the problem that hedging positions entered into earlier could lose their effectiveness as product prices change during the maturity of the contract. Thus, the firm continuously reevaluates and decides whether and how often to adjust its hedging position before expiration, or to wait and keep an existing contract and to enter into a new position (if any) upon expiration. Transaction costs complicate the firm's trade-offs further. On the one hand, longer-term contracts are more favorable because the firm does not have to replace its contracts very frequently. On the other hand, longer-term contracts are less flexible and incur transaction costs should the firm decide to terminate them before they expire. If transaction costs vary with the hedge ratio, the firm faces an additional trade-off between the benefits of higher hedging coverage and its higher costs. The idea that transaction costs are important determinants of risk management decisions is supported by empirical research such as Nance et al. (1993), Mian (1996), and Géczy et al. (1997), who find evidence consistent with economies of scale of risk management.

A number of insights arise from the model with respect to both the time-series and cross-sectional properties of risk management strategies. First, the model implies that in the presence of transaction costs, a non-monotonic relation exists between risk management activity and product price. For a firm with fixed debt, the optimal risk management strategy depends on the spot level of the firm's product price, relative to the firm's total costs. At very high prices, firms neither initiate new risk management contracts nor adjust existing contracts as financial distress is not

[3] Although not explicitly modelled, the framework can also accommodate costly external financing as in Froot et al. (1993) as an incentive to manage risk. The model does not directly incorporate other incentives to manage risk which are suggested by existing static theories such as, for example, information asymmetry between managers and investors (DeMarzo and Duffie, 1995), or managerial risk aversion (Smith and Stulz, 1985). Stulz (2002) provides a review of risk management rationales.

[4] The finite life of risk management instruments is most obvious where derivative securities are concerned. Few liquid derivative security markets (over the counter or exchange) offer maturities beyond ten years. Thus, the theoretical results are most applicable to risk management via derivative securities or other financial risk management contracts. However, arguably even other risk management tools, such as operational hedges, could have a maturity that is shorter than the firm's potentially infinite horizon and therefore eventually have to be replaced as their effects expire.

imminent and thus the expected benefits of risk management do not outweigh its transaction costs. As prices fall and financial distress becomes more likely, firms enter the active risk management zone in which they are more likely to initiate risk management contracts and actively replace and roll them over to avoid financial distress. However, as prices fall further, and firms become deeply distressed, they are again less likely to initiate or adjust risk management contracts. Cross-sectionally, the model suggests a similar non-monotonic relation between factors such as leverage, which affect the likelihood of financial distress, and risk management. While this non-monotonic relation has been suggested previously by Stulz (1996), this research is the first to explicitly generate the relation from a dynamic model.[5]

Second, the paper provides results regarding the maturity choice of risk management contracts. The model indicates that more deeply distressed firms tend to choose shorter maturities for newly initiated risk management instruments. Also, the model predicts that firms with higher transaction costs tend to change their risk management contracts less often and choose longer maturities. The results also imply that a firm with greater product price volatility chooses risk management contracts with longer maturity.

The structure of transaction costs also affects the hedge ratio of the firm's risk management contracts. The model suggests that, with zero costs or with fixed transaction costs, the firm always chooses the contract with the highest available hedge ratio as it offers maximum protection of the firm's cash flows. However, upon introduction of variable transaction costs that increase with the contract's hedge ratio, we observe variation in both the hedge ratio and the maturity of the chosen contract, indicating a trade-off between contract design and transaction costs.

With respect to the optimal adjustment of risk management instruments, we find that the optimal rollover strategy of risk management contracts is different from a mechanical replacement of expiring contracts. Optimal rollover and replacement decisions depend on the level of product prices and the features of risk management contracts already in place, regardless of whether the incentive for risk management is to reduce taxes or to reduce distress costs or both. For example, if we deal with a case in which the only motivation for risk management is the presence of distress costs, existing contracts are likely to be replaced before they mature if they are out-of-the-money. For a portfolio of forward contracts, this implies that the frequency of risk management contract adjustments should be higher during periods of rising spot prices than during periods of falling spot prices. For the case in which taxes and distress costs both serve as an incentive to manage risk, the adjustment dynamics are complex. However, regardless of the combination of risk management rationales, the features of existing contracts, such as moneyness and remaining maturity, are determined by the historic path of the spot price. Empirically, this implies that firms with identical characteristics observed at different times but with the same market conditions could have very different risk management contracts in place because the

[5]The non-monotonic relation is also consistent with evidence such as Mian (1996), Tufano (1996), and Géczy et al. (1997), who find only weak evidence that proxies of distress costs have a positive monotonic effect on corporate risk management.

observed firms could have reached a given state via different paths. Similarly, identical firms at the same current spot price could have differing risk management contracts in place, if their histories of firm-level characteristics (e.g., leverage) are different. Therefore, in some cases, empirical tests of risk management rationales could be improved by incorporating information about the preceding price and firm history.

The model parameters are calibrated to be consistent with empirical observations of firms in the gold mining industry. Specifically, parameter values are selected that roughly match the time-series properties of gold price returns and the financial ratios and production costs of gold mining firms during the sample period of 1993–1999, which is used for the empirical work. Given the calibrated parameters, we show that optimal risk management policies may have little impact on equity volatility, which is consistent with the aforementioned empirical findings. The reason is that, at a given time, only the portion of the firm's future cash flows, which occur during the risk management instruments' finite duration, can be hedged.

Several predictions of the model are tested using quarterly derivatives data for gold mining firms between 1993 and 1999. We employ a similar measure of risk management activity as in Tufano (1996) and Brown et al. (2001). The empirical tests regarding risk management activities do not provide a comprehensive analysis of all existing models, but instead focus on some of the new predictions motivated by our model. The empirical results show evidence of non-monotonicity between the measure of risk management activity and the likelihood of distress as proxied by several measures such as leverage and quick ratio. We also find a significant relation between gold spot prices and risk management activity as suggested by the model.

The tests also extend the existing empirical literature by contributing an analysis of maturity choice of risk management instruments. Based on a measure of the weighted-average maturity of a firm's risk management contracts, we find evidence of a non-monotonic relation between leverage and maturity consistent with the model's predictions.

The remainder of the paper proceeds as follows. Section 2 introduces the model. Solutions for the valuation of the firm's equity and the choice of the risk management strategy are provided in Section 3. Section 4 provides numerical results. Empirical evidence is presented in Section 5. Section 6 concludes.

2. Dynamic risk management theory

This section develops a continuous-time, infinite-horizon model of a firm that endogenously and dynamically adjusts its risk management contract, which is a function of the firm's exogenous product price. The model can be described by the following timeline.

At time 0

- The levered firm decides whether to initiate a risk management contract guaranteeing a set of forward prices for a chosen fraction (hedge ratio) of the firm's output and chooses its maturity.

Each subsequent time period

- The firm produces one unit of product at a fixed cost and realizes cash flows that are determined by the current spot price and the price guaranteed by the risk management contract (if any) and whether or not the firm is in financial distress.
- The firm can default, in which case the debtholders recover part of the firm's value and the equityholders get nothing and are obligated to terminate (pay out or cash out) any outstanding risk management contracts.
- If not in default, the firm meets its periodic debt payments and pays production costs and then makes a decision with respect to its risk management strategy:
  - the firm can enter a risk management contract and choose hedge ratio and maturity,
  - or if the firm currently operates with a risk management contract in place, it can choose to terminate the contract early by cashing out (or by paying out) its current position at a fair market value. Both the initiation and the termination of the risk management contract generate transaction costs.
- The residual cash flow after debt payments, taxes, and production costs is paid to the equityholders as dividends.

The firm is assumed to default on its debt optimally, i.e., when the market value of the firm's equity becomes zero. The firm's decisions with respect to the risk management strategy are made from the perspective of the shareholders who maximize the value of their equity stake.[6] Both equity and debt are priced fairly taking into account the risk management strategy of the equityholders. Because of a need to limit the dimensionality of the model, we are forced to make several modeling compromises. First, we do not allow the firm to change the structure of its debt over time. Second, we assume that the firm holds no cash, which implies that it pays all its residual cash flows as dividends.

2.1. Spot price and production costs

The firm continuously produces a unit of product, which can, for example, be viewed as a commodity, whose spot price p continuously evolves through time and is described by the log-normal process:[7]

$$\frac{\mathrm{d}p}{p} = (r - a)\,\mathrm{d}t + \sigma\,\mathrm{d}W_p, \tag{1}$$

where W_p is a Wiener process under the risk neutral measure Q; σ is the instantaneous volatility coefficient; r is the risk-free rate, which is assumed to be constant; and a, $(a \geqslant 0)$, is the convenience yield. The cost of production of one unit of product c, $(c \geqslant 0)$, is assumed to be constant. Revenue uncertainty driven by

[6] We can also consider the choice of the risk management policy from the perspective of all claimholders of the firm to maximize the total value of the firm's debt and equity.

[7] The model can easily be extended for any reasonable price process. For example, we can assume that the price follows a mean reverting process of the Ornstein–Uhlenbeck type or the two-factor process introduced in Gibson and Schwartz (1990).

variation in the output spot price is the only source of uncertainty explicitly modeled. In this sense, the results are most applicable to firms facing less cost uncertainty and more revenue uncertainty such as firms in many extraction industries (oil, gold, etc.) that have fairly predictable production costs.

The model assumes that the production quantity is non-stochastic. While quantity uncertainty could introduce interesting risk management dynamics, in particular with respect to the endogenous hedge ratio, its inclusion would make the numerical solution considerably more difficult and would require us to simplify other parts of the model. Static models of risk management that incorporate quantity risk include Koppenhaver (1985), Morgan et al. (1988), and Brown and Toft (2002).

2.2. Risk management contracts

At any time, the firm can choose to enter (or terminate, if any) a risk management contract that aims to reduce temporarily the risk related to the product's price uncertainty. The risk management contract guarantees a predetermined price for a chosen fraction h of the firm's product for the chosen maturity τ. Parameter h is the hedge ratio, which determines what portion of the cash flow is hedged by a chosen contract. When the firm enters a new risk management contract, the firm chooses the contract's hedge ratio among I available discrete choices $h \subseteq \{h_1 < h_2 < \cdots < h_i < h_{i+1} < \cdots < h_I\}$, where $0 \leqslant h_i \leqslant 100\%$ and h_I is the maximum available hedge ratio.[8] For simplicity, we assume that available hedge ratios are equally spaced with the step Δh.[9] In addition, the firm chooses the contract's maturity τ, $\tau \leqslant T$, where T is the maximum maturity available.

The risk management contract consists of a portfolio (continuum) of infinitesimally small (in terms of notional amount), fairly priced forward contracts with continuous maturities between zero and τ.[10] At the moment of origination, the risk management contract has zero expected value. Given the level of the spot price p_t at origination, the risk management contract guarantees the price of $p_t^* = p_t$ at the current time t, and in the next period $t + \Delta t$ the contract guarantees the price of $p_{t+\Delta t}^* = p_t\,\mathrm{e}^{(r-a)\Delta t}$, two periods from t the contract guarantees the price of $p_{t+2\Delta t}^* = p_t\,\mathrm{e}^{(r-a)2\Delta t}$, and so on until maturity. The entire contract guarantees the firm a price schedule for a fraction of its product $\{p_t^*, p_{t+\Delta t}^*, p_{t+2\Delta t}^*, \ldots, p_{t+\tau}^*\} = \{p_t, p_t\,\mathrm{e}^{(r-a)\Delta t}, p_t\,\mathrm{e}^{(r-a)2\Delta t}, \ldots, p_t\,\mathrm{e}^{(r-a)\tau}\}$ during the contract maturity τ, $(\tau \leqslant T)$. Thus, each risk management contract can be uniquely described by a tuple $\{p^*, h, \tau\}$, where h is

[8]While spot price uncertainty for the firm's product is the only source of uncertainty explicitly modeled, choosing a maximum available hedge ratio of less than 100% is a simple way to assess the effect of additional (non-hedgeable) background risk, which is not explicitly modeled in the current setting.

[9]Static models that incorporate the choice of the hedge ratio include Kerkvliet and Moffett (1991), and Froot et al. (1993).

[10]Alternatively, we can assume that the firm enters into a continuum of any reasonable derivative contracts including plain vanilla options. We believe that, if we consider a richer set of derivative instruments, it would not alter the qualitative results regarding the timing of the risk management contracts and the choice of maturity. Static models incorporating different payoff functions for the hedging contracts are Adam (2002a) and Brown and Toft (2002).

the chosen hedge ratio, p^* is the contract guaranteed price at which the firm is entitled to sell its product at the current time t, and τ is the time remaining in the contract before it matures. At a given time, p^* is sufficient to uniquely calculate the contract prices over the remaining maturity. Although, initially the contract has expected value of zero, its value (moneyness) fluctuates as the spot price changes and maturity gradually declines.

The market value of the risk management contract is the present value of the remaining cash flows. Thus the fair value of a risk management contract that has hedge ratio h and remaining maturity τ is $h\,V_t(p^*, p, \tau)$, where

$$V_t(p^*, p, \tau) = (p^* - p)\int_0^{\tau} \mathrm{e}^{(r-a)s-rs}\,\mathrm{d}s = \frac{(p^* - p)}{a}[1 - \mathrm{e}^{-a\tau}], \quad \tau \leqslant T, \tag{2}$$

where p is the spot price and p^* is the price at which, according to the contract, the firm is entitled to sell fraction h of its product at the current time t.

If not terminated earlier, the contract expires at its maturity. At maturity the firm can either enter a new contract at then prevailing forward prices or go on selling the product at the spot price for some time with a real option to enter a new contract at any time in the future. The firm, however, can also terminate an existing contract at any time prior to maturity. In that case, either the firm receives the fair value of the remaining cash flows associated with a contract $h\,V_t(p^*, p, \tau)$, if the contract is in the money (i.e., if $p^* > p$), or it has to pay the fair value for the contract, if it is out of the money (i.e., if $p^* < p$).

We assume that the firm has to pay transaction costs TC when it initiates a new contract and terminates a contract initiated earlier.[11] For the base case, we assume that the magnitude of transaction costs does not depend on the choice of the contract's hedge ratio. Such fixed transaction costs can occur because of fixed components of trading costs such as brokerage fees as well as because of fixed cost components of administering the risk management contract within the firm (e.g., internal reporting and accounting). In the numerical results, we also analyze variable transaction costs that depend on the hedge ratio.

The firm adjusts its risk management contracts in a discrete manner given that the transaction costs preclude the firm from performing continuous adjustment of its risk management position. One possible risk management strategy for the firm is to adjust and rollover its risk management contracts in the sense that the firm can always replace an existing risk management contract with a new contract of longer maturity by simultaneously terminating the current risk management contract and initiating a new one at the prevailing spot price. Hereby the firm faces a trade-off between incurring transaction costs and operating with a potentially suboptimal contract entered into earlier.

[11] Transaction costs of initiating the portfolio of futures contracts can be associated with margin requirements in the case of exchange-traded contracts. Bid-ask spreads and execution costs can also be viewed as a part of the overall transaction costs of a risk management program (see Ferguson and Mann, 2001).

Implicitly we assume that the firm can have only one active contract at a time. It would be more realistic to allow the firm to have multiple hedging contracts with overlapping maturities. However, this would result in a more complicated hedging structure. Therefore, to avoid the need for tracking multiple contracts and their hedge ratios and maturities, we assume that the firm adjusts its hedging position by terminating its current hedging contract and entering a new one.

2.3. Debt and firm cash flow

We assume that the firm issues a perpetual non-callable coupon bond. The amount of debt is exogenous and stationary, and the equityholders pay a continuous coupon d. We assume that the firm's cash flow after debt payments is taxed continuously at a constant corporate rate λ and that the periodic debt coupon payments d are tax-deductible. We also assume that there are no loss-offset or carry-forward provisions. The preceding assumption is a simple way of generating tax code convexity in the model.

The firm uses its income to meet its debt and tax obligation, with any residual being paid out as a dividend to the equityholders. The firm's income depends on whether or not the firm has a risk management contract. Thus, the firm's instantaneous dividend before taxes at any time t equals either $p - c - d$ if the firm has no risk management contract outstanding or $hp^* + (1-h)p - c - d$ otherwise, where p^* is the price according to the risk management contract originated earlier, h is the hedge ratio of the contract, and c is the constant cost of production of one unit of product. The firm's instantaneous tax obligation equals

$$(\lambda)\max[0, p' - c - d] \geqslant 0, \tag{3}$$

where d represents the periodic debt payments and p' equals either the spot price p_t (if the firm does not have an outstanding risk management contract) or $p' = hp^* + (1-h)p$, otherwise.

If there is insufficient cash flow to meet the debt payment, the firm can raise capital by issuing equity. In practice, a firm could retain part of its cash flow and then use it for future debt service, which could affect the risk management strategy and the valuation of equity. Similarly, we ignore the option to store the product and to time the sale of the product. Although feasible, incorporating these features would lead to an increase in the dimensionality of the model and would further complicate the analysis.

Although the debt's tax shield function can provide a motivation for its use in the capital structure even in the presence of distress costs, we do not explicitly model the trade-off between tax shields and distress costs, which could be used to determine optimal leverage endogenously. Instead we assume a fixed amount of debt as part of the capital structure. While this assumption of static debt is not uncommon in the risk management literature, it ignores potential interactions between capital structure and risk management decisions. In principle, the model can be extended by allowing equityholders to change the level of debt dynamically as in Mauer and Triantis (1994) or in Titman and Tsyplakov (2002). However, endogenizing capital structure in such a fashion detracts from the focus of the analysis of the dynamic features of risk management and significantly complicates the numerical procedure.

2.4. Financial distress

When the firm's instantaneous cash flow cannot cover the debt payments, the firm is required to issue equity. We assume that the firm can costlessly issue equity only if the firm is well capitalized. By assumption, the firm is well capitalized if its leverage ratio $D/(D+E)$ does not exceed a certain critical level L, where D and E are market values of debt and equity, respectively. However, if the firm's leverage is above the critical level L and the cash flows cannot cover debt payments, the firm experiences financial distress. We assume that in financial distress the firm incurs additional cash flow losses because customers, suppliers, or strategic partners may not be willing to deal with financially distressed companies. Unlike default costs that are incurred by debtholders at bankruptcy, distress costs are directly borne by equityholders. These costs are important because they may be incurred long before bankruptcy is imminent and they provide incentives to manage risk. The magnitude of financial distress costs in the model is determined by how low the firm's cash flow falls relative to the debt payments and production costs. In particular, we assume that financial distress costs are proportional to the difference between the firm's required debt payments and its income net of production costs. The firm incurs distress costs equal to

$$C^P_{\text{Distress}}\,\delta\left[\frac{D}{E+D}-L\right]\max[0,\ -p'+c+d], \tag{4}$$

where $\delta[x]=1$ if $x>0$ and $\delta[x]=0$ otherwise, C^P_{Distress} is constant, and p' equals either the spot price p_t or, $p'=hp^*+(1-h)p$.

Financial distress, as modeled here, does not create any permanent damage to the firm but causes temporary cash flow loss. In other words, the distress situation does not directly affect the future productivity of the firm. Allowing for permanent damage would require us to keep track of the duration of distress, which would increase the dimensionality of the problem.

2.5. Default and bankruptcy

The firm defaults optimally (incorporating the value of the risk management contract) when the value of its equity is zero. We assume that in the event of default the equityholders get nothing and the debtholders recover the value U of the unlevered firm minus default costs DC proportional to U, i.e., at default the debt value satisfies $D(p)=(1-DC)U(p)$. For simplicity, we assume that the unlevered firm has no access to risk management and that the unlevered firm can permanently shut down its operations when the price drops sufficiently below the costs. The price at which the unlevered firm shuts down its operations is endogenously determined.[12] We assume that at default the equityholders are obligated to terminate (pay out or cash out) an outstanding risk management contract $\{p^*,h,\tau\}$ at a fair market price,

[12]In the context of the model, given debt in place, what is assigned as the value of the firm at default does not affect the risk management strategy. However, this assumption could affect the pricing of the debt.

$h\,V_t(p^*, p, \tau)$, where $V_t(p^*, p, \tau)$ is the value of the contract as described in Eq. (2). The last assumption specifies that the counterparties of the risk management contract never default on their contract payments. Later results show that the firm (behaving optimally) never reaches the default boundary while holding an out-of-the-money contract. Thus the assumption effectively requires only that the other counterparty be without default risk. This is a typical assumption of most theoretical models including Stulz (1984), Smith and Stulz (1985), and Froot et al. (1993). An exception is the work by Cooper and Mello (1999), who assume that the pricing of the hedging contracts incorporates a premium (spread) that reflects the level of default risk associated with the firm seeking to hedge. As a result, in their model, the terms in the hedging contract affect the choice of the hedging strategy.

3. Valuation

The valuation of equity and debt both depend on the firm's risk management strategy. Because we assume complete markets for the firm's product, debt and equity can be regarded as tradable financial claims for which the usual pricing conditions must hold. Effectively, the model assumes that the information about the product prices and the risk management strategy of the firm is publicly available. This section discusses the valuation of equity for the levered firm. The valuation of the unlevered firm and the numerical solution algorithm are discussed in the appendix.

The equity value $\mathrm{E} = \mathrm{E}(p, p^*, h, \tau)$ is the net present value of the cash flows to shareholders that depend on four state variables, which include the spot price p, the price p^* of the firm's current risk management contract, the contract hedge ratio h, and the contract's remaining maturity τ. The values can be determined by solving stochastic control problems with free boundary conditions, where the control variable is the decision variable $i = \mathrm{i}(p, p^*, h, \tau) \in \{\{h, \tau\}, 0, -1\}$ that describes the firm's decision either to initiate the risk management contract, to keep its risk management position unchanged, or to terminate an existing contract. If $\mathrm{i}(p) = \{h, \tau\}$, the firm initiates the contract that has hedge ratio h and maturity $\tau, h \subseteq \{h_1 < h_2 < \cdots < h_i < h_{i+1} < \cdots < h_I\}$ and $0 < \tau \leqslant T$; if $\mathrm{i}(p, p^*, h, \tau) = -1$, the firm terminates the outstanding contract $\{p^*, h, \tau\}$; and if $\mathrm{i}(p, p^*, h, \tau) = 0$, the firm keeps its position unchanged. The decision to terminate the contract depends upon all four state variables, while the decision to initiate the contract depends only on the spot price, since without an existing contract the firm is in the state $(p, p^*, h, 0)$ for any p^* and h. In the states where $\tau = 0$ (i.e., no remaining maturity of the contract), the firm does not have a contract outstanding, and the equity value satisfies $\mathrm{E}(p, p^*, h, 0) = \mathrm{E}(p, p^{*\prime}, h', 0)$ for any p^*, $p^{*\prime}$, h, and h'. At the states where $\tau > 0$, the firm has a contract outstanding whose value depends on the hedge ratio h and the current price p^* guaranteed by the contract.

In each state (p, p^*, h, τ), the shareholders choose their risk management strategy as well as their default policy to maximize the market value of their equity $\mathrm{E}(p, p^*, h, \tau)$. Using standard arbitrage arguments and accounting for the transaction cost of initiating and terminating the risk management contract, the value of the equity is given by the solution to the following stochastic control problem:

$$\max_{i\in\{\{h,\tau\},0\}}\Bigg[\frac{1}{2}\,\sigma^2p^2\mathrm{E}_{pp}+(r-a)p\mathrm{E}_p-r\mathrm{E}+p-c-d$$
$$-(\lambda)\max[0,\ p-c-d]-C^P_{\text{Distress}}\delta\left[\frac{D}{E+D}-L\right]\max[0,\ -p+c+d]\Bigg]=0, \tag{5}$$

at time $\tau=0$ or at any time thereafter if the firm has no risk management contract in place or

$$\max_{i\in\{-1,0\}}\Bigg[\frac{1}{2}\,\sigma^2p^2\mathrm{E}_{pp}+(r-a)p\mathrm{E}_p+(r-a)p^*\mathrm{E}_{p^*}-r\mathrm{E}-\mathrm{E}_\tau$$
$$+hp^*+(1-h)p-c-d-(\lambda)\max[0,\ hp^*+(1-h)p-c-d]$$
$$-C^P_{\text{Distress}}\delta\left[\frac{D}{E+D}-L\right]\max[0,\ -hp^*-(1-h)p+c+d]\Bigg]=0, \tag{6}$$

at time $0<\tau\leqslant T$ if the firm has an active risk management contract $\{p^*,h,\tau\}$.[13] The term $-E_\tau$ in Eq. (6) represents a linear decrease in the remaining maturity of the outstanding risk management contract as time progresses. Because we are dealing with an infinite horizon model, the value of equity is independent of time, i.e., $\mathrm{E}_t(p,p^*,h,\tau)=0$.

The equity value satisfies a set of free-boundary and smooth pasting conditions. Denote p^1 as the market price at which the firm optimally initiates a risk management contract with hedge ratio h and maturity τ, i.e. $\mathrm{i}(p^1)=\{h,\tau\}$. The free boundary condition at which the firm initiates a contract with maturity τ is

$$\mathrm{E}(p^1,p^*,h,0)=\mathrm{E}(p^1,p^1,h,\tau)-TC\quad\text{for any } p^*, \tag{7}$$

where TC are the transaction costs introduced earlier. Denote $p^0(h^0,p^{*0},\tau^0)$ as the price at which the firm terminates its current contract $\{p^{*0},h^0,\tau^0\}$, i.e., $\mathrm{i}(p^0,p^{*0},p,\tau^0)=-1$. At the boundary at which the firm terminates its current contract $\{p^{*0},\tau^0\}$, the equity value satisfies

$$\mathrm{E}(p^0,p^{*0},h^0,\tau^0)=\mathrm{E}(p^0,p^*,h,0)-TC+h^0\,V(p^0,p^{*0},\tau^0)\quad\text{for any } p^* \text{ and } h, \tag{8}$$

where $h^0\,V(p^0,p^{*0},\tau^0)$ is the fair value of the outstanding contract $\{p^{*0},h^0,\tau^0\}$ given market price p^0, as calculated in Eq. (2).

In the state where an existing contract expires $\{p^*,h,\varepsilon\}$, i.e., $\varepsilon\xrightarrow{+}0$, one has the following boundary condition:

$$\mathrm{E}(p,p^*,h,\varepsilon)\to\mathrm{E}(p,p^*,h,0)\quad\text{as } \varepsilon\xrightarrow{+}0,\ \text{for any } p \text{ and } p^*. \tag{9}$$

One also needs to impose the free boundary condition, which ensures that the equity value is greater or equal to zero for any firm that has no outstanding contract.

$$\mathrm{E}(p,p^*,h,0)\geqslant 0. \tag{10}$$

[13]Subscripted equity values denote partial derivatives.

In the default region one needs to consider two cases: (1) the firm defaults without any risk management contract outstanding, and (2) the firm defaults with an outstanding contract. In the latter case, the model assumes that the equityholders are forced to terminate the contract $\{p^{*0}, h^0, \tau^0\}$ at a fair market price and pay (or receive) the proceeds.

$$\mathrm{E}(p^0, p^{*0}, h^0, \tau^0) = h^0\, V(p^0, p^{*0}, \tau^0) \quad \text{for } 0 < \tau^0 < T \tag{11}$$

4. Numerical results

We calibrate the model to match the data used in the empirical work. We then characterize the firm's optimal risk management dynamics based on the numerical solution and provide comparative statics.

4.1. Calibration and parameter values

The model is calibrated to match empirical observations for firms in the gold mining industry, which are described in more detail in Section 5. In particular, the calibration seeks to replicate a firm that continuously produces one ounce of gold per year. The parameters of the spot price are calibrated to the gold prices observed during the period between January 1992 and December 2000. For this period, the daily COMEX gold closing prices obtained from Bloomberg fluctuate between \$242 and \$410 per ounce, with an average price of \$346 per ounce. Given that the model assumes that the firm produces one unit of product per year, in the numerical simulations, the price value is varied within a given range with an initial $p = \$345$/ounce. Volatility is calculated for daily, monthly, and quarterly gold price returns and equals 11.8%, 10.4%, and 12.1%, respectively. For the base case parameter values, the volatility of the spot price σ is set at 10%.

The 12-month lease rate for gold is used as a proxy for the convenience yield of gold. The average lease rate for gold obtained from Bloomberg for February 1995 to January 2000 is 2.04%, therefore the value for the convenience yield a is set at 2%.[14] For the same period, the rate of the U.S. three-month Treasury bill fluctuates between 6.05% and 2.85%. For the numerical analysis the risk-free rate is set to $r = 4.0\%$.

To parametrize the level of production costs, we use the data reported in Tufano (1996). Given his sample of more than 90 U.S. and Canadian gold mining firms, he shows that the average (median) cost is between \$239 and \$243 per ounce (\$235–\$239/ounce) with a standard deviation across firms of \$58 per ounce. Therefore, for the base case parameter values, production costs of one ounce of gold are approximated at $c = \$250$/ounce.

[14] Schwartz (1997) reports similar numbers in his calibration approach for spot and futures prices as well as for the average convenience yield. See also Fama and French (1988).

The parametrization of the level of the coupon payment d requires an analysis of the level of short-term and long-term debt obligations of the firms in the gold mining industry. Compustat quick ratios in the sample vary between 0.1 and 16.2, with an average of 2.9 and a median of 2.1, while the empirically observed mean leverage ratio is 17% (median is 18%) with a standard deviation of 13%. We use the observed quick ratio as a proxy for the ratio of net income to debt payments, which in the model is represented by the ratio of net income to coupon payment, $(p-c)/d$. Given the assumption in the model that the firm produces one unit (one ounce) of gold, we roughly match the observed quick ratios, by setting the level of $d = \$50$ so that the model generates a similar quick ratio, i.e., $(p-c)/d = (\$345-250)/\$50$ is close to 2. Given the above level of debt payments and other base case parameter values, the leverage generated by the model is 11%, which is within the range of the empirically observed leverage ratios for gold mining firms.

We assume that the critical leverage L at which the firm begins to incur distress costs is $L = 20\%$. Approximately one-fifth of the observations of firm-quarters in the empirical sample have leverage values in excess of 20%. The parameters of distress costs are difficult to estimate because one cannot directly observe the cash flow losses that would be attributable to a distress situation. Opler and Titman (1994) show that, during industry downturns, more highly leveraged firms experience a drop in equity values that is more than 10% greater than the drop experienced by less highly levered firms. A 10% difference in the drop of equity values implies several hundred percent in temporary cash flow losses in distress situations. For the base case parameter values, the proportional distress costs C^P_{Distress} are initially set at 200% of the difference between required debt payments and the firm's income.

The maximum available hedge ratio h_I is also difficult to parametrize, because in reality it is not known what fraction of uncertain cash flows can be hedged by the firm. Given the data on selected gold mining firms, one observes that a proxy of the hedge ratio varies across firms from 0% to a maximum of 42% (a detailed description of the empirical estimate is given in Section 5). For the base case parameter values, we set the parameter h_I at 50%, which means that the maximum available hedge ratio is 50%. We assume that the firm can have five different choices of hedge ratios ($I = 5$): $h \subseteq \{10\%, 20\%, 30\%, 40\%, 50\%\}$. In the data set of gold mining firms the maximum average reported maturity of the risk management contracts is 4.6 years. Thus, in the base case, the maximum maturity T is chosen to equal five years.

For the calibration of the transaction costs TC of contract initiation and termination, we refer to values documented in the literature. Huang and Masulis (1999) and Ferguson and Mann (2001) report that, for commodity futures, transaction costs associated with the bid-ask spread vary between less than 1 b.p. to up to 15 b.p. as a fraction of the notional value of the contract. Transaction costs can also arise at the firm level as a result of the costs of running the risk management strategy. Therefore, for the base case, TC is set at \$6, which is less than 35 b.p. of the value of the contract to deliver one unit of commodity for five years at a typical price of \$345 (nominal value of the contract $= 5 \times \$345 = \$1,725$). The tax rate is set at $\lambda = 40\%$. Bankruptcy costs DC are set at 0.75. All base case parameter values are reported in Table 1.

Table 1
Base case parameter values for the numerical solution of the model

Parameter	Value
σ, volatility of the product price	10%
T, maximum maturity of the risk management contract	5 years
α, convenience yield	2%/year
r, risk-free rate	4%/year
C^P_{Distress}, proportional costs of financial distress	200%
TC, transaction costs of the RM contract	\$6
h, hedge ratio of the RM contract	10%, 20%, 30%, 40%, 50%
C, production costs	\$250/year
d, debt payment	\$50/year
L, critical leverage for distress	20%

4.2. Financial distress, taxes, transaction costs, and risk management

We introduce various imperfections and assess how they affect risk management strategy. To understand the marginal impact of each imperfection, we discuss results for seven different scenarios depending on the absence/presence of each imperfection. We consider three sources of imperfections: financial distress costs, taxes, and risk management transaction costs. If present in a particular scenario, each variable is calibrated to the value of the base case discussed previously. Table 2 shows the firm's decision to default on its debt and/or to enter into a new risk management contract as a function of the spot price (given that the firm does not have a contract in place or that the contract in place is at expiration). Entering into new contracts is indicated by showing the contract's maturity, while empty table cells indicate that the firm does not enter into a new contract. We do not show the hedge ratio, because the firm always chooses the maximum available hedge ratio $h_I = 50\%$ for all scenarios. This result is expected as the firm is always better off with a higher hedge ratio given no transaction costs or fixed transaction costs that do not vary with the hedge ratio. In a later section we provide model results for transaction costs that increase with the hedge ratio of the contract in which case the choice of the hedge ratio becomes nontrivial.

4.2.1. Risk management without transaction costs

Scenario 1 is a frictionless world without financial distress costs, taxes, or transaction costs. In such a frictionless setting, we obtain the familiar result that risk management is irrelevant as it does not affect firm value.[15]

The next three scenarios, 2–4, assume that risk management contracts are free of transaction costs and that there are imperfections in the form of financial distress costs and taxes, which are discussed both individually and jointly. Scenario 2 contains distress costs at the base case parameter value of 200%, and scenario 3

[15]The firm defaults optimally at very low spot prices (\$120 given our parameter values) when the value of equity equals zero.

Table 2
Financial distress, taxes, transaction costs, and risk management
The table reports the firm's decision to initiate a new risk management contract (given no contract in place) and the choice of contract maturity (in years) as a function of the current spot price p. The hedge ratio choice (not shown) always equals the maximum available hedge ratio of 50% for all contracts. The dash indicates that the firm does not initiate a new contract. "Default" indicates that the firm defaults for the given price level. The risk management decision is reported for seven scenarios, which vary according to the absence or presence of distress costs, transaction costs, and taxes. "No" corresponds to the case with zero value for the parameter, and "Yes" corresponds to the parameter value at base case level. The remaining parameter values are as in the base case reported in Table 1.

Scenario	1	2	3	4	5	6	7
Distress costs	No	Yes	No	Yes	Yes	No	Yes
Taxes	No	No	Yes	Yes	No	Yes	Yes
Transaction costs	No	No	No	No	Yes	Yes	Yes
Spot price	Risk management						
120	Default	Default	Default	Default	Default	Default	Default
135	Irrelevant	Default	Default	Default	Default	Default	Default
150	Irrelevant	Default	Default	Default	Default	Default	Default
165	Irrelevant	Default	5.0	Default	Default	—	Default
180	Irrelevant	5.0	5.0	Default	—	—	Default
195	Irrelevant	5.0	5.0	5.0	—	—	—
210	Irrelevant	5.0	5.0	5.0	4.7	—	—
225	Irrelevant	5.0	5.0	5.0	5.0	—	4.7
240	Irrelevant	5.0	5.0	5.0	5.0	—	5.0
255	Irrelevant	5.0	5.0	5.0	5.0	—	5.0
270	Irrelevant	5.0	5.0	5.0	5.0	—	5.0
285	Irrelevant	5.0	5.0	5.0	5.0	5.0	5.0
300	Irrelevant	5.0	5.0	5.0	5.0	5.0	5.0
315	Irrelevant	5.0	5.0	5.0	—	5.0	5.0
330	Irrelevant	5.0	5.0	5.0	—	5.0	5.0
345	Irrelevant	5.0	5.0	5.0	—	5.0	—
360	Irrelevant	5.0	5.0	5.0	—	—	—

contains taxes at the base case tax rate of 40%. Scenario 4 is the joint case. Both distress costs as well as taxes give the firm an incentive to manage risk. For these scenarios without transaction costs, the firm always seeks maximum protection in terms of duration and amount of cash flows protected from distress costs and/or tax code convexity and thus uses both maximum maturity and maximum hedge ratio contracts.

In scenario 2, risk management contracts enable the firm to reduce or avoid financial distress costs that occur at low spot prices. Given that there are no transaction costs, the firm always initiates a new contract if the existing contract is terminated or expires for all spot prices above the default boundary. The intuition of the result is straightforward. At high spot prices above distress levels, the risk management contract temporarily protects the firm from incurring distress cost should the price drop, while at distressed prices such contracts are still optimal because they protect the firm from even higher distress costs, which would be incurred if spot prices fall further in the future.

In scenario 3, given the non-symmetric (convex) nature of taxes, the firm is always better off having a risk management contract in place to lower expected taxes. The risk management contract protects the firm from outcomes, in which prices rise in the future leading to higher taxes, while the firm can costlessly terminate and replace any existing risk management contract, if spot prices decline thereby lowering tax payments. This strategy is especially valuable for spot prices near the boundary between positive and negative income (before taxes) where the tax function exhibits the highest convexity. This result is in line with the static model of Smith and Stulz (1985).

The dynamics of risk management adjustment for the case with taxes are different from the case with financial distress costs. With taxes, contracts are replaced, if spot prices decline below the contract price, while rising prices lead to contract replacement in the case of distress costs. Thus, while both financial distress costs and taxes provide incentives to manage risk and give rise to the same new contract initiation strategy with respect to hedge ratio, maturity choice, and the spot prices at which contracts are initiated, the adjustment of existing contracts in response to spot price changes is different between the two incentives.

Given that, with financial distress costs or taxes, the firm always initiates contracts with maximum maturity and hedge ratio at all spot prices, we expect to see a similar initiation strategy when both imperfections occur together, which is what we find in scenario 4. However, given the differences in the adjustment of existing contracts, the combined adjustment strategy is complex for the case with both imperfections.[16]

4.2.2. Risk management with transaction costs

Scenarios 5–7 contain transaction costs in addition to the other two imperfections with all parameters at their base case values. As in the previous cases without transaction costs, the introduction of fixed transaction costs results in the use of contracts with the maximum available hedge ratio whenever the firm employs risk management contracts. However, as the results show, the introduction of transaction costs leads to significant changes with respect to the spot prices at which risk management contracts are initiated, contract maturity, and adjustment strategy.

We first discuss distress costs and taxes separately before combining all three imperfections. We begin with scenario 5, which has distress costs and transaction costs but no taxes. To better illustrate the intuition, Table 3 provides a detailed presentation of scenario 5, which adds information regarding financial distress zones and leverage ratios for different spot prices. Given the base case parameter values, the firm experiences a cash flow short fall for spot prices below \$300 and enters

[16] We also find that the default boundary varies both compared with the frictionless case and between the three scenarios with imperfections. Both distress costs as well as taxes lead to ''earlier'' default at a higher spot price, because both imperfections reduce the value of equity, ceteris paribus. Combining the two imperfections in scenario 4 increases the default boundary further.

Table 3
The base case of the risk management strategy
Column 5 reports the firm's decision to initiate a new risk management contract (given no contract in place) as a function of the current spot price p (column 1). Columns 6 and 7 report the choice of hedge ratio and contract maturity in years, respectively. The table also shows interest coverage ratio (column 2), leverage (column 3), price zones for initiation or no initiation of new contracts (column 8), and the price zones for financial distress and default (column 4). The distress zone is the price range in which the product price is below the sum of production costs and debt payments and leverage is above its critical level. The interest coverage ratio is the ratio of net income to debt payments $(p-c)/d$. The parameter values are as in the base case reported in Table 1.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Spot price	Interest coverage ratio	Lever-age (in %)	Default/ distress/ no distress	Initiate new RM contract?	Hedge ratio (in %)	Maturity of new RM contract	RM zones
165	−1.7	100.0	Default				
180	−1.4	71.1	Distress	No			Zone 3
195	−1.1	48.5	Distress	No			
210	−0.8	35.7	Distress	Yes	50	4.7	
225	−0.5	28.1	Distress	Yes	50	5.0	
240	−0.2	23.2	Distress	Yes	50	5.0	
255	0.1	19.8	No distress	Yes	50	5.0	Zone 2
270	0.4	17.4	No distress	Yes	50	5.0	
285	0.7	15.7	No distress	Yes	50	5.0	
300	1.0	14.3	No distress	Yes	50	5.0	
315	1.3	13.1	No distress	No			
330	1.6	12.1	No distress	No			
345	1.9	11.3	No distress	No			Zone 1
360	2.2	10.6	No distress	No			

financial distress for spot prices below \$240 at which point leverage exceeds the critical level. The firm optimally defaults, if the spot price falls below \$165.

Unlike in the scenario without transaction costs, the firm does not use risk management contracts for high prices or very low (highly distressed) prices. Thus the decision to initiate a risk management contract has a non-monotonic relation with the level of the spot price. A firm without a risk management contract in place immediately initiates a risk management contract if the price lies within the range of \$210–300. For prices outside this range the firm does not initiate a new risk management contract. Thus the model predicts three distinct zones: no new contract for high prices (zone 1), new contract initiation (zone 2) for the middle range of prices and then again no new contract (zone 3) as spot prices decrease further. These zones refer to the decision to initiate a new risk management contract. However, because of transaction costs, firms do not necessarily terminate existing risk management contracts as soon as spot prices move out of the new contract initiation zone.

The intuition behind the non-monotonic relation between initiation and spot price is that for very high prices (zone 1), the probability of distress is low, and thus the firm does not initiate a risk management contract because it incurs transaction costs and the contract is likely to expire before the price declines to the distress zone. As the spot price declines, the probability of distress increases, and at some point the firm initiates the risk management contract, because the transaction costs are more than compensated by the smaller expected (distress related) cash losses.[17] As the price declines and leverage increases above the critical level, cash losses stemming from distress increase and so does the probability of bankruptcy. As a result, at prices near the default boundary, the firm has less incentive to initiate the risk management contract because it would reduce (at least temporally) the value of the default option because of the transaction costs of contract termination, and thus risk management would benefit the debtholders at the expense of the equityholders. This last result is in line with the asset substitution problem identified by Jensen and Meckling (1976) suggesting that a firm near bankruptcy has an incentive to increase the volatility of its assets.

The non-monotonic relation between spot price and risk management also has important cross-sectional implications via a change in parameter d, the level of debt payments: an increase in the level of debt payments leads to a parallel shift of the zones up and down along the price scale. This result is straightforward because changes in the debt level imply parallel shifts of the distress zone and the default boundary as well. Therefore, given the same spot price, firms with different debt payments (as determined by leverage) would make different decisions with respect to the initiation of new risk management contracts.

Table 3 also indicates that, given the base case parameter values, the firm chooses risk management contracts with the maximum available maturity of five years in most parts of zone 2. However, as the price approaches the lower end of the zone, the firm chooses shorter maturity contracts. This result is driven by the default option, which becomes increasingly important at lower prices: at a price that puts the firm into distress (but not yet default), a high probability of early termination exists for a contract with long maturity either because the price could drop further and default occurs, which triggers early contract termination, or because the price could increase and the firm terminates the existing risk management contract early to replace it with a new one, which locks in higher forward prices. In both cases the firm incurs transaction costs that can be avoided by using a shorter maturity contract, which may expire (without transaction costs) before early termination becomes necessary.

[17]The firm initiates the contract before leverage increases to the critical level. This result can be explained by the fact that the risk management contract locks in the price for only 50% of the firm's cash flows while the other 50% is still vulnerable to price fluctuations.

The fact that, with the exception of very distressed firms, the maturity of new contracts is chosen near or at the maximum available maturity implies that one should observe demand for long-term risk management contracts. However, many derivatives markets either do not exist or are highly illiquid for long maturities. This is most likely the result of limited supply for such contracts. The idea that derivatives markets become less liquid as maturity increases could be incorporated into the model by allowing the transaction costs to vary with maturity.

Scenario 6, which contains taxes and transaction costs, provides similar results to the previous scenario with financial distress costs. As in scenario 5, we observe three risk management zones. At very high and very low spot prices the expected benefits of risk management do not outweigh its transaction costs. This is because the tax function exhibits very little convexity at high or low prices. While both scenario 5 and 6 give rise to three risk management zones, the zones do not necessarily occur at the same spot prices all else equal. Furthermore, the default boundary is located at slightly different prices for the two scenarios.

We include all three imperfections in scenario 7.[18] Similar to the previous two scenarios with transaction costs, we observe three risk management zones such that no new contracts are initiated at high prices far from the distress boundary and at low (very distressed) prices. Both the upper and the lower bound of the risk management zone are determined by the interaction of distress costs, taxes, and transaction costs. In principle, it is also possible to observe two separate (disjointed) active risk management zones. This is the case if the spot price at which taxable income becomes non-positive is sufficiently far away from the spot price at which the distress-triggering critical leverage level is exceeded. In this situation, the firm tends to use risk management to reduce taxes in a higher price zone in which taxes are most convex, while the firm uses risk management in a lower price zone to avoid or reduce distress costs.

4.3. Additional analysis of risk management with financial distress and transaction costs

We present a detailed discussion of the risk management strategy for scenario 5, which considers distress costs and transaction costs but does not consider taxes. We choose scenario 5 for the following reasons. As mentioned previously, imperfections such as taxes and distress costs both provide an incentive for risk management. However, as shown by the analysis, the interaction of taxes and distress costs introduces considerable additional complexity to the risk management strategy and thus would make it more difficult to interpret some of the already fairly complex dynamic results that follow. Thus, we use scenario 5 as a base case to analyze the adjustment of risk management contracts (rollover strategy), the evolution of risk management contracts as spot prices change, the effect of risk management on firm value and equity volatility, the relation between hedge ratio choice and variable

[18] As in the case without transaction costs, the inclusion of both taxes and distress costs increases the default boundary.

transaction costs, and the comparative statics with respect to the initial parameter values.

4.3.1. Adjustment of risk management contracts

The previous results for the base case with financial distress and transaction costs establish that a firm always initiates a new risk management contract in zone 2 given that no contract is in place at the time. Thus expiring contracts are replaced immediately in zone 2. However, the preceding discussion of maturity choice already alluded to the fact that the firm's risk management strategy is by no means static or mechanical within zone 2. The firm's decision to incur transaction costs for early termination and replacement of an existing contract depends on the characteristics (forward prices and remaining maturity) of the contract in place and the current spot price, which determines the forward prices available from new contracts. Ceteris paribus, if the current spot price is high relative to the price guaranteed by the contract, the firm is more willing to replace the existing contract. In these cases the firm is also more willing to replace the existing contract, the longer its remaining maturity, while very short-term contracts are not replaced early as they expire costlessly in the near future.

Table 4 analyzes the firm's adjustment strategy of risk management contracts as a function of the current spot price p, the spot price p^* at initiation of the existing contract, and the remaining maturity τ of the existing contract. Specifically, for various combinations of p and p^* (measuring the moneyness of the existing contract), the table shows the range of remaining maturity within which an existing contract is replaced; empty inputs in the table indicate that the existing contract is not replaced. For example, as the table shows, the firm terminates contracts initiated at a price of \$225, if the spot price increases to the level of \$270 and the contract remaining maturity is greater than 0.60 years, or if the price increases to \$255 or above and the remaining maturity is longer than 0.35.

Table 4 indicates that within zone 2 the firm often terminates and replaces out-of-the-money risk management contracts. The propensity to replace as proxied by τ increases as contracts are further out of the money. Intuitively, within zone 2, the firm wants to lock in at higher prices and get rid of lower price contracts to lessen the additional cash losses resulting from distress (given that leverage is still above the critical level), even though the contract termination incurs transaction costs. But, if the price either declines or does not increase enough, the firm keeps the contract until maturity and then immediately enters a new contract if the spot price is still within zone 2. Thus, the model predicts that the firm keeps its risk management contract until it matures if it is in the money and tends to terminate the risk management contract before maturity if it is out of the money. Empirically, these results imply that, conditionally on observing the firm in zone 2, periods of price increases should lead to more frequent adjustments to risk management contracts, while these adjustments should be less frequent during periods of price decreases.

Table 4 also contains results regarding the firm's risk management activities at prices close to the default boundary. As established previously, a firm without a risk

Table 4
Adjustment of risk management contracts

Inputs of the table are the range of remaining maturity τ of the existing contracts for which the firm terminates (and possibly replaces) the contract given that the contract was originated at price p^* and the current spot price is p. Various combinations of p and p^* measure the moneyness of the existing contract. The dash indicates that the existing contract is not terminated. Contracts terminated at spot prices above the default boundary of \$165 are immediately replaced with a new contract, while contracts terminated at or below the default boundary are not replaced as the firm immediately defaults subsequent to termination. As an example, the firm replaces contracts initiated at a price of \$225, if the spot price increases to the level of \$270 or above and the contract's remaining maturity is greater than 0.60 years, or if the price increases to \$285 or above and the remaining maturity is longer than 1.45 years. The parameter values are as in the base case reported in Table 1.

Current spot price, p	Spot price at initiation of existing contract, p^*						
	210	225	240	255	270	285	300
165	$0.05<\tau$	$0.10<\tau$	$0.10<\tau$	$0.10<\tau<1.65$	$0.15<\tau<0.75$	—	—
180	—	—	—	—	—	—	—
195	—	—	—	—	—	—	—
210	—	—	—	—	—	—	—
225	$0.70<\tau$	—	—	—	—	—	—
240	$0.25<\tau$	$0.60<\tau$	—	—	—	—	—
255	$0.35<\tau$	$0.35<\tau$	$0.45<\tau$	—	—	—	—
270	$0.65<\tau$	$0.60<\tau$	$0.60<\tau$	$0.60<\tau<3.05$	$0.70<\tau<1.60$	—	—
285	$1.45<\tau$	$1.45<\tau$	$1.45<\tau$	$1.45<\tau$	$1.45<\tau<2.40$	—	—
300	—	—	—	—	—	—	—

management contract in place defaults optimally at a spot price of \$165. We argue above that firms without a risk management contract in place do not initiate new contracts at low prices close to the default boundary because risk management lowers the value of the default option. The results in this section show that firms not only refrain from initiating new contracts but may even actively terminate existing contracts, akin to asset substitution, as default becomes imminent. For example, a firm with a risk management contract originated at \$255 terminates (without replacement) the contract at the default spot price of \$165 and immediately defaults provided that the remaining maturity is between 0.10 and 1.65 years. The intuition for termination without replacement followed by default is that the equityholders can realize the fair value of the in-the-money contract. However, if the contract price is originated at sufficiently high prices and if the remaining maturity is sufficiently long or sufficiently short ($0.15<\tau<1.65$ years in the example), the firm does not terminate and default. For long maturities, spot prices could improve over the remaining life of the contract, thereby avoiding default altogether, while for short maturities the value of the contract is insufficient to warrant the transaction costs of early termination.

4.3.2. Evolution of risk management contracts and spot price history

The firm always holds a risk management contract in zone 2. Furthermore, the firm frequently terminates and replaces risk management contracts within zone 2. However, even outside zone 2, one can still observe the firm with a risk management contract initiated earlier while the price was in zone 2, if the spot price subsequently drifts away from zone 2 during the maturity of the contract. Therefore, this section considers the evolution of the firm's risk management position. In particular, we analyze the probability that the firm is observed with a risk management contract and the remaining average maturity of the observed contract. The remaining maturity of an existing risk management contract is not the same as the maturity choice of a newly initiated contract discussed previously.

Generally, one expects the probability of observing the firm with a risk management contract to decline as the price moves away from zone 2 either above or below. To verify the above statement we simulate ten-year spot price paths, while recording the characteristics of the risk management contract (if any) for each price.[19] Specifically, for each price level on the simulated path, we calculate the probability of observing the firm with an unexpired risk management contract and the average maturity of the observed contract conditional on the contract being in place. Because the price level at which the path starts affects the observed probability, the simulations are repeated for three different starting spot price levels, \$195, \$270, and \$345, which represent price levels in zones 3, 2, and 1, respectively.

The simulation results, reported in Table 5, confirm the previous intuition that for a given price outside zone 2 one can observe otherwise identical firms at different times that at the same price level have different risk management contracts in place. The reason for this is that the firms have reached the same price level via different paths and some of them still have remaining contracts initiated earlier, while for others all contracts initiated earlier have expired. As indicated in Table 5, the calculated probability and the average remaining maturities of the risk management contracts vary with the starting price level of the simulations given the same current spot price level. These results imply that information about the current spot price is not always sufficient to uniquely predict whether a firm has risk a management contract in place and to predict the observed characteristics of the contract. For example, when the current spot price is \$345, the probability of observing a firm with a risk management contract in place is 12% when the starting spot price is also \$345. For the same current spot price of \$345, but a starting spot price of \$270, the probability of observing a firm with a risk management contract is much higher at 84%. Similarly, the average observed maturity differs by approximately six months between the two cases. To adequately predict the optimal risk management strategy of the firm, one may need to have information about the past time series of the price.

[19]Specifically, 500,000 simulated paths are run. For the simulations the drift of the spot price is adjusted from the risk neutral to the real measure to match the empirically observed drift of the gold price of 9.9% during 1970–2000. While simulating the adjusted stochastic process, we keep track of the hedging contract of the firm and incorporate its hedging strategy calculated in (5) and (6). If at any time the simulated price reaches the default boundary, the path is terminated and a new path is started.

Table 5
Evolution of risk management contracts and spot price history

The table reports the results of spot price simulations and the observed risk management contracts along the simulated price paths. For each spot price level on the path (column 1), we report the probability *prob* (in percent) of observing the firm with a risk management contract and the average remaining maturity $\overline{\tau}$ (in years) of the observed contract conditional on the contract being in place (columns 3–8). The simulation results are reported for three different starting spot price levels: \$345, \$270, and \$195 (columns 3–4, 5–6, and 7–8, respectively). The results are based on 500,000 simulated paths for each starting point. The drift of the spot price is adjusted from the risk neutral to the real measure to match the empirically observed drift of the gold prices of 9.9% during 1970–2000. If, at any time, the simulated price reaches the default boundary, the path is terminated and a new path is started. The table also shows the initiation decision and maturity choice for new contracts (given no contract in place). The parameter values are as in the base case reported in Table 1.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Spot	Maturity of	Starting spot price for simulation					
price	new risk management	345		270		195	
	contract	*prob*	$\overline{\tau}$	*prob*	$\overline{\tau}$	*prob*	$\overline{\tau}$
165	Default	0	0.0	0	0.0	0	0.0
180	No new contract	82	2.03	83	1.93	76	3.05
195	No new contract	91	2.29	93	2.21	78	3.49
210	4.7	100	2.47	100	2.47	100	3.86
225	5.0	100	2.48	100	2.66	100	3.99
240	5.0	100	2.59	100	2.95	100	4.00
255	5.0	100	2.78	100	3.36	100	3.99
270	5.0	100	3.05	100	3.80	100	3.93
285	5.0	100	3.40	100	3.83	100	3.83
300	5.0	100	3.81	100	3.43	100	3.42
315	No new contract	53	3.54	96	3.02	97	3.03
330	No new contract	25	3.17	90	2.66	93	2.69
345	No new contract	12	2.83	84	2.36	88	2.40
360	No new contract	8	2.53	77	2.10	84	2.15
375	No new contract	7	2.28	70	1.89	78	1.93
390	No new contract	6	2.06	63	1.70	73	1.75
405	No new contract	6	1.87	56	1.54	68	1.59
420	No new contract	5	1.71	50	1.41	63	1.45
435	No new contract	5	1.57	44	1.30	58	1.34

The results also suggest that even if two firms are exposed to identical price paths and currently have identical leverage, they might still exhibit different risk management strategies if their leverage histories, and hence their distress and default boundary histories differ. Path-dependency also applies to the cases in which risk management is motivated by different rationales such as taxes.

The simulation results also have an interesting cross-sectional implication with respect to observed maturity. As shown in Table 5, risk management contracts observed outside zone 2 tend to have shorter remaining maturities than contracts inside zone 2. This is intuitive because contracts outside zone 2 are old contracts, which the firm does not terminate because of transaction costs. Therefore, one can relate observed maturity to cross-sectional variation in leverage: at a given spot price, firms with low, medium, and high leverage may be observed in zones 3, 2, and 1 and therefore should exhibit short, long, and short observed remaining maturity of their risk management contracts, respectively. The results in Table 5 also reveal that the average remaining maturity of the risk management contract is the longest, if the firm is close to the distress boundary. This implies that near the distress boundary the firm more frequently replaces its risk management contracts before they mature and thus is observed with fresh contracts having long remaining maturities.

4.3.3. Value creation, equity exposure, and risk management

To measure the impact of risk management on firm value, the volatility of equity, and credit spreads, this section analyzes a firm that has no access to risk management contracts, for example, because of prohibitively high transaction costs. Table 6 reports the above measures for the base case firm with access to risk management and each measure's respective difference from the value observed for a firm without access to risk management. As expected, firm value declines with spot prices and is higher for the firm with access to risk management. Furthermore, the difference in firm value increases as the spot price declines. This result is straightforward because the impact of risk management becomes more important for lower spot prices when the firm is less profitable. This implication is supported by empirical work by Simkins et al. (2004), who suggest that the value of hedging is more important during economic downturns. As the results show, credit spreads also increase as spot prices fall and default becomes more likely. The reduction in credit spreads resulting from risk management is relatively small as it never exceeds 0.5 basis points, which is in line with the fact that the default boundary for the firm that has access to risk management is almost the same as for the firm without such access.

One can also measure the extent to which the availability of risk management can affect spot price exposure and reduce the volatility of equity returns. We compare the instantaneous equity volatility of the firm with and without access to risk management.[20] As expected, the results show that the reduction of equity volatility is greater for lower prices. However, the economic contribution appears to be relatively small. For example, at a price of $240 (distress zone), risk management can reduce equity volatility only by approximately 2%. The reasons for such a small decrease in volatility are twofold: the firm can reduce the uncertainty of its cash flows only for a limited maturity, while the uncertainty of the income to be received after the maximum maturity cannot be reduced at all. The unhedged cash flows after the maximum maturity are more uncertain and contribute a bigger portion to the overall volatility of equity, even though the later cash flows are discounted more.

[20] Based on Ito's lemma, the instantaneous volatility of equity is given by $\sigma p(\mathrm{E}_p/E)$, where the subscripted equity values denote partial derivatives.

Table 6
Value creation, equity exposure, and risk management

Columns 3–6 show leverage (in percent), firm value (in thousands), credit spread (in basis points), and equity volatility (in percent), as a function of the spot price (column 1) for a firm that has access to risk management contracts. Columns 7–10 report the difference (in percent with the exception of the credit spread, which is in basis points) of each respective measure compared with a firm without access to risk management contracts. Column 2 shows the initiation decision and maturity choice in years for new risk management contracts. The hedge ratio (not shown) equals the maximum available hedge ratio of 50% for all contracts. The dash in column 2 indicates that the firm does not initiate a new contract. "Default" indicates that the firm defaults for the given price level. The parameter values are as in the base case reported in Table 1.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Spot price	Maturity	Firm with option to manage risk				Impact of option to manage risk			
		Leverage	Firm value	Credit spread	Equity volatility	Leverage	Firm value	Credit spread	Equity volatility
165	Default	100.0							
180	—	71.1	1.1	247	188	−5.9	7.8	−.22	−30
195	—	48.5	1.9	142	101	−4.1	7.8	−.28	−9
210	4.7	35.7	2.9	90	68	−2.7	7.1	−.32	−5
225	5.0	28.1	3.9	61	51	−1.8	6.2	−.36	−3
240	5.0	23.2	4.9	43	41	−1.3	5.3	−.39	−2
255	5.0	19.8	5.9	31	34	−0.9	4.6	−.44	−3
270	5.0	17.4	6.8	23	28	−0.6	3.2	−.34	−3
285	5.0	15.7	7.6	18	25	−0.3	2.2	−.25	−2
300	5.0	14.3	8.5	14	22	−0.2	1.5	−.16	−1
315	—	13.1	9.3	11	21	−0.1	1.1	−.11	−1
330	—	12.1	10.1	8	20	−0.1	.8	−.09	−1
345	—	11.3	10.9	7	19	−0.1	.6	−.07	−0.5
360	—	10.6	11.6	5	18	−0.0	.4	−.04	−0.4

4.3.4. *Hedge ratio choice with variable transaction costs*

The firm always chooses the maximum hedge ratio available if the transaction costs are fixed over different levels of the hedge ratio. We expect that the endogenous choice of the hedge ratio could yield non-trivial results once we allow for variable transaction costs that increase with the hedge ratio. For variable transaction costs, the firm faces a trade-off between transaction costs and risk management coverage. Thus, we might observe that the firm chooses lower hedge ratios to save on transaction costs. To explore this issue, we analyze three variable transaction cost functions such that costs increase either linearly, quadratically, or cubically as the hedge ratio increases ($TC = v_1 * h$, $TC = v_2 * h^2$, and $TC = v_3 * h^3$). We parameterize each transaction cost function using v_i such that the cost of the maximum hedge ratio is equal to the base case fixed cost of \$6.

Table 7 provides information similar to Table 3 (decision to initiate, hedge ratio, and maturity of new contracts given that no contract is in place) for fixed transaction

Table 7
Hedge ratio choice with variable transaction costs

Panel A shows the transaction costs *TC* of initiating or terminating risk management contracts under four cost functions that depend on the hedge ratio *h*: fixed, linear, quadratic, and cubic ($TC = v_0$, $TC = v_1 * h$, $TC = v_2 * h^2$ and $TC = v_3 * h^3$). We parameterize each transaction cost function using v_i such that the cost of the maximum hedge ratio is equal to the base case fixed cost of \$6 ($v_0 = 6$, $v_1 = 12$, $v_2 = 24$ and $v_3 = 48$). For each transaction cost function, Panel B reports the firm's decision to initiate a new risk management contract (given no contract in place), the chosen hedge ratio (HR), and the maturity (MAT) of the new contract (in years) as a function of the current spot price *p*. The dash in Panel B indicates that the firm does not initiate a new contract. "Def." indicates that the firm defaults for the given price level. With the exception of transaction costs, the parameter values are as in the base case reported in Table 1.

Panel A: Transaction costs

Hedge ratio	Fixed costs	Linear costs	Quadratic costs	Cubic costs
10%	6.00	1.20	0.24	.05
20%	6.00	2.40	0.96	.38
30%	6.00	3.60	2.16	1.30
40%	6.00	4.80	3.84	3.07
50%	6.00	6.00	6.00	6.00

Panel B: Risk management

Spot price	HR	MAT	HR	MAT	HR	MAT	HR	MAT
165	Def.	Def.	Def.	Def.	Def.	Def.	Def.	Def.
180	—	—	—	—	10%	2.8	20%	2.5
195	—	—	—	—	20%	3.5	30%	3.1
210	50%	4.7	50%	4.7	40%	4.4	50%	4.1
225	50%	5.0	50%	5.0	50%	4.9	50%	4.5
240	50%	5.0	50%	5.0	50%	5.0	50%	4.7
255	50%	5.0	50%	5.0	50%	5.0	50%	5.0
270	50%	5.0	50%	5.0	50%	5.0	50%	5.0
285	50%	5.0	50%	5.0	50%	5.0	50%	5.0
300	50%	5.0	50%	5.0	50%	5.0	50%	5.0
315	—	—	—	—	—	—	—	—
330	—	—	—	—	—	—	—	—
345	—	—	—	—	—	—	—	—

costs (base case) and for the three different variable transaction cost functions. While the linear transaction cost function yields the same risk management strategy as the one obtained under fixed transaction costs, we find that both quadratic and cubic transaction cost functions yield different risk management strategies with non-trivial results regarding the hedge ratio. For both quadratic and cubic transaction costs, we find that the firm chooses lower and thereby cheaper hedge ratios and shorter maturities as spot prices decline toward the default boundary. Furthermore, the firm uses risk management contracts right up to the point of default, whereas, with fixed or linear transaction costs, the firm does not use risk management contracts at all for very low (highly distressed) prices, which are referred to as zone 3 in Table 3. With quadratic or cubic costs, the firm uses shorter-maturity, cheap contracts with low hedge ratios in zone 3. Our intuition for this result is that, with relatively cheap (in terms of transaction costs) contracts, the firm can essentially fine-tune the trade-off between the expected costs of risk management (sum of transaction costs and reduction in the value of the default option) and the expected benefits of risk management (reduction of distress costs and avoidance of default). The results suggest that transaction costs can affect not only the maturity and the timing of the risk management strategy but also the choice of the hedge ratio. This implies that the level and structure of transaction costs can be an important determinant of risk management activity.

4.3.5. Comparative statics

We examine how changing the initial parameter values affects the firm's risk management strategy. In each case, one parameter is varied while all other parameters are held at the level in the base case. Given the base case assumption of fixed transaction costs, the results show that the firm always chooses the maximum available hedge ratio for all parameter values considered in the comparative statics. However, the risk management zones and maturity choices are affected by changes in the parameters.

Table 8 provides risk management results for different fixed transaction cost levels. We find that firms with higher fixed transaction costs tend to have a narrower price zone in which they initiate risk management contracts and tend to choose longer maturities for the risk management contracts. This result is in line with empirical studies such as Dolde (1993), Nance et al. (1993), Mian (1996), and Géczy et al. (1997) showing that smaller firms, which arguably incur higher costs associated with maintaining risk management programs, are more often observed without any risk management contracts in place compared with bigger firms in the same industry.

An increase in the value of parameter h_I implies that the firm can hedge a greater fraction of its cash flow using the maximum hedge ratio contract, which is always employed if transaction costs are fixed. As shown in Table 9, an increase in h_I results in a narrower price range within which the firm initiates risk management contracts because the firm can wait longer before initiating risk management contracts. Moreover, a higher h_I parameter implies that the firm tends to choose shorter maturities.

An empirical implication of this result can be based on the idea that smaller firms and more homogeneous firms, which tend to have less variety of products, are exposed to fewer risks. Such firms may be able to hedge a greater fraction of their cash flows using a particular risk management contract. Thus, the empirical interpretation of the model is the following: the probability of observing a firm with risk management contracts is lower for firm types that can hedge a greater fraction of cash flows with a single risk management contract. However, the model does not consider multiple sources of uncertainty, which could be less than perfectly correlated giving less homogeneous firms a natural diversification benefit thereby reducing the need of such firms to use risk management contracts.

Table 8
Comparative statics for transaction costs
Columns 2–6 report choices of maturity in years of new hedging contracts for firms with different levels of transaction costs, TC, as a function of the current spot price (column 1). The hedge ratio (not shown) equals the maximum available hedge ratio of 50% for all contracts. "No new contract" indicates that the firm does not initiate a new contract for the given price level. "Default" indicates that the firm defaults for the given price level. The other parameter values are as in the base case reported in Table 1.

(1)	(2)	(3)	(4)	(5)	(6)
	$TC = 2$	$TC = 4$	$TC = 6$	$TC = 10$	$TC = 20$
Spot price	Maturity of new risk management contract				
165	Default	Default	Default	Default	Default
180	No new contract	No new contract	No new contract	No new contract	No new contract
195	3.3	4.0	No new contract	No new contract	No new contract
210	3.6	4.3	4.7	5.0	No new contract
225	3.9	4.6	5.0	5.0	5.0
240	4.2	4.8	5.0	5.0	5.0
255	5.0	5.0	5.0	5.0	5.0
270	5.0	5.0	5.0	5.0	5.0
285	5.0	5.0	5.0	5.0	5.0
300	5.0	5.0	5.0	No new contract	No new contract
315	5.0	No new contract	No new contract	No new contract	No new contract
330	No new contract	No new contract	No new contract	No new contract	No new contract
345	No new contract	No new contract	No new contract	No new contract	No new contract

Table 10 shows that an increase in the distress costs C^P_{Distress} implies a narrower zone 3, which is the lower-price part of the distress zone in which the firm does not initiate risk management contracts. Also, firms tend to pick shorter maturity contracts. Intuitively, firms that loose a greater portion of cash flows in distress tend to use their risk management contracts more often within the distress zone. In addition, such firms tend to replace and rollover their contracts more frequently, which results in a selection of shorter-term contracts. Thus, industries with high distress costs should exhibit shorter-maturity hedging programs.

As reported in Table 11, an increase in the volatility of the spot price implies that the firm employs risk management contracts over a wider range of spot prices. Also, because of an increase in the value of the default option, the critical price level at which the firm defaults decreases as the volatility increases. For the same reason, zone 3 (the distressed risk management zone without new contract initiation) tends to widen with volatility implying that, for higher volatility, the value of the default option exceeds the value of the option to reduce risk. Thus, the model predicts that firms tend to intensify their risk management programs during periods of higher expected uncertainty, which is in line with the empirical findings of Naik and Yadav (2003), who show that dealers in the United Kingdom Treasury market hedge to a greater extent during periods of higher economic uncertainty.

Table 9
Comparative statics for maximum hedge ratio
Columns 2–6 report choices of maturity in years of new hedging contracts for firms with different levels of maximum hedge ratio parameter, h_I, as a function of the current spot price (column 1). The hedge ratio (not shown) equals the maximum available hedge ratio for all contracts. "No new contract" indicates that the firm does not initiate a new contract for the given price level. "Default" indicates that the firm defaults for the given price level. The other parameter values are as in the base case reported in Table 1.

(1)	(2)	(3)	(4)	(5)	(6)
	$h_I = 10\%$	$h_I = 25\%$	$h_I = 50\%$	$h_I = 75\%$	$h_I = 100\%$
Spot price	Maturity of new risk management contract				
165	Default	Default	Default	Default	Default
180	No new contract	No new contract	No new contract	No new contract	No new contract
195	No new contract	No new contract	No new contract	4.0	3.7
210	No new contract	No new contract	4.7	4.2	4.1
225	No new contract	5.0	5.0	4.6	4.5
240	No new contract	5.0	5.0	5.0	4.8
255	5.0	5.0	5.0	5.0	5.0
270	5.0	5.0	5.0	5.0	5.0
285	5.0	5.0	5.0	5.0	5.0
300	No new contract	5.0	5.0	5.0	No new contract
315	No new contract	No new contract	No new contract	No new contract	No new contract
330	No new contract	No new contract	No new contract	No new contract	No new contract
345	No new contract	No new contract	No new contract	No new contract	No new contract

The results also imply that a firm with greater volatility of its product price chooses risk management contracts with longer maturity. Intuition for this result can be gained by analyzing the rollover strategy of risk management contracts. Additional results (not shown) indicate that the firm with greater price volatility tends to wait for a greater price change before terminating an existing contract. Specifically, such a firm terminates an existing contract and immediately enters a new contract, only if the spot price increases by a significantly higher amount than in the cases with lower volatility. Therefore, a firm exposed to higher volatility tends to prefer longer maturity contracts.

5. Empirical evidence

We test several predictions derived from the model with a particular focus on the time-series properties and the predicted non-monotonicity of risk management activities and on risk management maturity choice.[21]

[21]We also perform an analysis along the lines of Petersen and Thiagarajan (2000), confirming the limited impact of risk management on equity exposure to variations in gold prices. The results are not shown in the paper but are available from the authors upon request.

Table 10
Comparative statics for distress costs

Columns 2–6 report choices of maturity in years of new hedging contracts for firms with different levels of proportional distress costs, C^P_{Distress}, as a function of the current spot price (column 1). The hedge ratio (not shown) equals the maximum available hedge ratio of 50% for all contracts. "No new contract" indicates that the firm does not initiate a new contract for the given price level. "Default" indicates that the firm defaults for the given price level. The other parameter values are as in the base case reported in Table 1.

(1)	(2)	(3)	(4)	(5)	(6)
	$C^P_{\text{Distress}} = 0.5$	$C^P_{\text{Distress}} = 0.75$	$C^P_{\text{Distress}} = 2$	$C^P_{\text{Distress}} = 3$	$C^P_{\text{Distress}} = 4$
Spot price	Maturity of new risk management contract				
135	Default	Default	Default	Default	Default
150	No new contract	Default	Default	Default	Default
165	No new contract	No new contract	Default	Default	Default
180	No new contract	No new contract	No new contract	Default	Default
195	No new contract	No new contract	No new contract	No new contract	No new contract
210	5.0	5.0	4.7	4.1	3.8
225	5.0	5.0	5.0	4.4	4.0
240	5.0	5.0	5.0	4.8	4.3
255	5.0	5.0	5.0	5.0	4.5
270	5.0	5.0	5.0	5.0	5.0
285	No new contract	No new contract	5.0	5.0	5.0
300	No new contract	No new contract	5.0	5.0	5.0
315	No new contract	No new contract	No new contract	5.0	5.0
330	No new contract	No new contract	No new contract	No new contract	No new contract
345	No new contract	No new contract	No new contract	No new contract	No new contract

5.1. Data

The empirical analysis employs survey data from the gold mining industry, which are used in several previous studies such as Tufano (1996), Brown et al. (2001), and Adam (2002b). [22] The 100 companies that are included in the data set are publicly traded gold producers based in the United States and Canada. The data contain quarterly information from the first quarter of 1993 through the third quarter of 1999 on the risk management instruments held by these firms, including the amount of the firms expected future production and specific information regarding each of the firms hedging positions, for example, the strike price and approximate maturity of each option. A detailed description of this data set is provided in Tufano (1996).

Following Tufano (1996), delta (Δ) is computed for each option position, deltas of negative one are assigned to all other short positions, and deltas of positive one are assigned to all other long positions.[23] The hedged volume HV of each hedging

[22] The raw data are provided by Ted Reeve, a financial analyst for Scotia Capital Markets.

[23] We assume that all options mature on the final day of the period in which they are classified in the survey. The delta for an option contact is calculated using the Black–Scholes model. Volatility is based on

Table 11
Comparative statics for spot price volatility
Columns 2–6 report choices of maturity in years of new hedging contracts for firms with different levels of spot price volatility, σ, as a function of the current spot price (column 1). The hedge ratio (not shown) equals the maximum available hedge ratio of 50% for all contracts. "No new contract" indicates that the firm does not initiate a new contract for the given price level. "Default" indicates that the firm defaults for the given price level. The other parameter values are as in the base case reported in Table 1.

(1)	(2)	(3)	(4)	(5)	(6)
	$\sigma = 6\%$	$\sigma = 8\%$	$\sigma = 10\%$	$\sigma = 12\%$	$\sigma = 14\%$
Spot price	Maturity of new risk management contract				
150	Default	Default	Default	Default	Default
165	Default	Default	Default	No new contract	No new contract
180	Default	No new contract	No new contract	No new contract	No new contract
195	No new contract	4.4	No new contract	4.7	4.8
210	4.4	4.7	4.7	5.0	5.0
225	4.6	4.9	5.0	5.0	5.0
240	4.7	5.0	5.0	5.0	5.0
255	4.8	5.0	5.0	5.0	5.0
270	5.0	5.0	5.0	5.0	5.0
285	5.0	5.0	5.0	5.0	5.0
300	No new contract	5.0	5.0	5.0	5.0
315	No new contract	No new contract	No new contract	5.0	5.0
330	No new contract	No new contract	No new contract	No new contract	5.0
345	No new contract	No new contract	No new contract	No new contract	No new contract

position is then computed as the product of delta and notional volume NV. Each quarter hedged volume is summed over all hedge positions and divided by the sum of forecasted production $PROD$ to arrive at the hedging delta-percentage DP. The hedging positions are categorized into annual maturity buckets of up to five years. Our model views risk management as a set of discrete choices of initiation and termination of risk management contracts. In the data, one does not necessarily expect to observe such discrete changes in the risk management strategy. Rather one expects the intensity of risk management as measured by DP to vary with market conditions and firm characteristics as predicted by the model.

To test the model's predictions with respect to the firm's hedge maturity choice, the average maturity MAT of the firm's hedge position is computed using hedged volume as weights. To assign maturities it is assumed that each hedge position matures on the final day of the period in which it is classified in the survey. Both the

(*footnote continued*)
the annualized standard deviation of gold prices for the previous 90 trading days. The price of gold is the closing price on COMEX from Bloomberg. Risk-free rates are from Bloomberg. Gold lease rates are from Bloomberg and Kitco.

measure of risk management activity *DP* and the measure of risk management maturity *MAT* describe the characteristics of observed risk management contracts rather than the initiation and cancellation of risk management contracts.

For each firm quarter in the survey, the following data items are obtained from Compustat: market value of equity, total assets, quick ratio, total debt, and the Z-score for the likelihood of financial distress as introduced by Altman (1968). All data items are quarterly with the exception of the Z-score, which has annual frequency. Leverage is computed as total debt divided by total assets.

Compustat data are available for 51 of the 100 companies, which appear at least once in the survey. Compustat firms appear on average in 17 of 27 survey quarters, while companies not on Compustat appear on average in 12 of 27 survey quarters. Because we intend to study the time-series properties of risk management activities, 15 of the 51 Compustat firms are excluded, because they appear in fewer than 12 survey quarters, which may bias the sample to larger, more successful firms. The remaining 36 firms constitute the data set for the empirical work.

Summary statistics are shown in Table 12. The mean and median hedging delta-percentage are 15% and 12%, respectively. Both values are comparable to the results of Brown et al. (2001), who use a sample similar to ours. Mean and median maturity of the hedging instruments is 1.6 years. Both hedging delta-percentage and maturity exhibit considerable time-series variation as measured by the cross-sectional average of each variable's within-firm time-series standard deviation. Mean and median leverage are 18% and 17%, respectively, with considerable cross-sectional variation, which is also present for quick ratios and Z-scores.

5.2. Univariate results

One testable implication of the model is the non-monotonic relation between risk management activities and the likelihood of financial distress. The model predicts that firms deep in financial distress and firms far away from financial distress have less incentives to reduce risk than firms between the two extremes.

To test this prediction of non-monotonicity cross-sectionally, we use leverage, the quick ratio, and the Compustat Z-score as measures for the likelihood of financial distress. For leverage and the quick ratio, we divide all observed firm quarters into three equal-sized groups (proxying low, medium, and high likelihood of distress). For the Z-score, firm quarters are assigned to the three groups based on the cut-offs of 1.81 and 3.00 used by Altman (1968). As shown in Table 13, these sorts generate dispersion in the measures across the groups.

We then compare risk management activity as proxied by average hedging delta-percentages for the three groups based on each of the measures. The average hedging delta-percentage increases when moving from the low likelihood of distress group to the medium likelihood of distress group regardless of the measure used. Based on t-tests, the difference between the first two subsets is significant at the 1% level for the leverage and quick ratio sorts, and significant at the 10% level for the Z-score sorts. More important, there is also evidence of the non-monotonic relation predicted by the model. For the leverage and quick ratio sorts, hedging delta-percentages decrease

Table 12
Descriptive statistics for gold mining firms

The table shows sample mean, median, standard deviation, minimum, and maximum for 36 U.S. and Canadian gold mining firms. Data are quarterly (except Z-score) from January 1993 to March 1999. For each firm, quarters are averaged; the resulting averages are used to calculate the statistics shown across firms. Hedging delta-percentage is the total delta-adjusted volume of risk management contracts divided by forecasted production. Standard deviation of delta-percentage is the standard deviation of the hedging delta-percentage across firm quarters. Change in hedging delta-percentage is calculated as the quarter-to-quarter absolute value of the change in the hedging delta-percentage, which is not due to changes in production forecasts or changes in option deltas. Maturity (in years) is the average remaining maturity of hedging contracts weighted by delta-adjusted volume. Market value is the value of the firm's equity (in $M). Production (in 1,000 ounces) is the firm's forecasted gold production over the next four to five years. Assets (in $M) is the firm's total assets. Debt (in $M) is the firm's total debt. Leverage is the ratio of total debt to total assets. Z-score is a measure of bankruptcy prediction as in Altman (1968) observed at the end of the previous year. Hedging and production data are from the Global Gold Hedge Survey. All other data are from Compustat.

	Mean	Median	Standard deviation	Minimum	Maximum
Quarters in survey	21	23	6	12	27
Delta-percentage	15%	12%	12%	0%	42%
Standard deviation delta-percentage	11%	10%	7%	0%	30%
Change in delta-percentage	5%	4%	4%	0%	20%
Maturity	1.6	1.6	0.6	0.5	2.6
Standard deviation maturity	0.5	0.5	0.3	0.2	1.2
Market value	804	343	1,264	9	4,784
Production	477	215	692	12	2,931
Assets	669	263	995	7	4,011
Debt	185	26	349	0	1,414
Leverage	18%	17%	13%	1%	48%
Quick ratio	2.9	2.1	3.0	0.1	16.2
Z-score	4.3	2.8	5.5	−0.4	29.2

as one moves from the medium likelihood of distress group to the high likelihood of distress group. The magnitudes of the decreases are 5% and 3%, respectively. The former decrease is significant at the 1% level, and the latter is significant at the 5% level. For the Z-score sorts, there is a statistically insignificant increase in the hedging delta-percentage.

The observed evidence of a non-monotonic relation between leverage and risk management activity sheds light on results in previous studies such as Nance et al. (1993) and Tufano (1996), which do not find supportive evidence when testing for the positive monotonic relation predicted by a static view of risk management.

The model suggests that, holding a firm's costs constant, there should be a non-monotonic relation between hedging delta-percentages and spot prices. As the spot price decreases toward the firm's costs, one expects firms to intensify (initiate) risk management, suggesting a higher hedging delta-percentage. However, as the spot price falls significantly below the firm's costs, one expects the relation to reverse as firms use less risk management again.

Table 13
Univariate statistics for risk management and financial distress

The table shows average hedging delta-percentages, *t*-tests, and number of observations for subsets of firm quarters in the gold mining firm sample. Subsets are formed by ranking firm quarters on three measures for the likelihood of financial distress: leverage, quick ratio, and *Z*-score. Averages and maxima for the measures are also shown. For leverage and quick ratio, subsets are formed by dividing all quarters with valid observations for the measure into three equal-sized groups. For the *Z*-score, firm quarters are assigned based on the cut-offs of 1.81 and 3.00 used by Altman (1968). Hedging data are from the Global Gold Hedge Survey. All other data are from Compustat. Data are quarterly (except *Z*-score) from January 1993 to March 1999. *** Significant at 1%; ** Significant at 5%; * Significant at 10%.

		Likelihood of distress		
		Low	Medium	High
Leverage	Mean	2%	18%	39%
	Max	11%	25%	112%
Delta-percentage	Mean	9%	21%	16%
	t-test	7.97***		2.68***
	Observations	215	240	223
Quick ratio	Mean	6.8	1.8	0.3
	Max	54.6	2.7	1.0
Delta-percentage	Mean	11%	19%	16%
	t-test	5.38***		1.76**
	Observations	233	242	233
Z-score	Mean	9.03	2.45	0.48
	Max	68.30	2.98	1.78
Delta-percentage	Mean	14%	15%	18%
	t-test	1.30*		(1.28)
	Observations	267	198	260

Observing this non-monotonic relation between spot prices and hedging delta-percentages in the data is complicated by the following two issues. First, firms may not maintain constant costs during the sample period (e.g., changes in leverage cause changes in interest expense), which would shift the spot price at which the relation reverses. Second, even if costs are relatively stable over time, one may not have enough variation in spot prices during the sample period to observe the same firm in the three different zones. In other words, firms with very low costs (in zone 1) could exhibit no relation between spot price and hedging delta-percentage as their risk management activities are not affected by financial distress considerations. Firms with medium costs could exhibit a purely negative relation between spot price and hedging delta-percentage as a spot price increase (decrease) moves them from zone 2 (1) to zone 1 (2). Firms with high costs could exhibit a purely positive relation between spot price and hedging delta-percentage as a spot price increase (decrease) moves them from zone 3 (2) to zone 2 (3).

Table 14
Univariate statistics for risk management and spot prices

The table shows average hedging delta-percentages, *t*-tests, and number of observations for subsets of firm quarters in the gold mining firm sample. Subsets are formed by independent sorts on average firm leverage (across all firm quarters) and average spot price of gold during the quarter. Observations are split into two approximately equal-sized groups based on the spot price. Observations are split into three groups based on average firm leverage allocating approximately 10–15% of all observations into the top and bottom leverage category. Hedging data are from the Global Gold Hedge Survey. Spot gold prices are from Datastream. All other data are from Compustat. All other data are from Compustat. Data are quarterly from January 1993 to March 1999. *** Significant at 1%.

		Price	
Leverage (%)		< $358/oz.	> 358/oz.
0–5	Mean delta-percentage	10.5%	12.5%
	t-test	−0.97	
	Observations	68	42
5–40	Mean delta-percentage	19.3%	13.7%
	t-test	3.71***	
	Observations	285	309
>40	Mean delta-percentage	10.6%	9.6%
	t-test	0.42	
	Observations	24	42

To investigate the above issues, the firms in the sample are split into subsets based on a proxy of costs. In Table 14, firms are split based on their average leverage (proxying for interest expense). We hypothesize that firms with extremely low leverage exhibit only a weak relation between spot prices and hedging delta-percentages and that firms with extremely high leverage exhibit a positive relation between spot prices and hedging delta-percentages. The remaining firms with medium leverage should exhibit a negative relation between spot prices and hedging delta-percentages. Cut-offs for leverage are chosen such that approximately 10–15% of firms are in either extreme group. The sample quarters are then split evenly based on the average spot price during the quarter. Supporting the suggested non-monotonicity, the results indicate no relation between hedging delta-percentages and spot prices for firms with extremely low leverage, and a negative relation between hedging delta-percentages and spot prices in the middle group. Firms with extremely high leverage do not exhibit a significant relation, which could stem from the small sample size.[24]

[24]As a robustness check, we also estimate a time-series regression (results not shown) for each firm in which we regress the delta-percentage on the average spot price during the quarter, the square of the average spot price, and the volatility of the spot price (calculated as the annualized standard deviation of daily gold returns over the 90 days preceding the end of the quarter). For most regressions we find a negative relation between average spot prices and delta-percentages.

5.3. Panel results for risk management activity

We report regression tests for the model's predictions regarding risk management activity as proxied by hedging delta-percentages. Because the focus of the analysis is on the time-series properties of hedging delta-percentages and the non-monotonic relation between measures of the likelihood of financial distress and hedging delta-percentages, the regressions do not include other variables previously suggested in the literature, which explain cross-sectional variation in hedging delta-percentages.[25] Thus we estimate a fixed effects panel regression model allowing the intercepts to pick up potentially unexplained firm-specific variation. Specifically, the following specification is estimated for firm i and quarter t:

$$DP_{it} = c_i + \beta_1 Spot_t + \beta_2 Vol_t + \beta_3 MV_{it} + \beta_4 Lev_{it} + \beta_5 Lev_{it}^2 + \varepsilon_{it}, \tag{12}$$

where DP is the hedging delta-percentage, $Spot$ is the average spot gold price (in \$/oz.) during the quarter, Vol is the annualized standard deviation of daily gold price returns during the quarter, MV is the market value (in \$1,000,000) of the firm's equity at the end of the quarter, Lev is the firm's leverage measured as the ratio of debt to assets, Lev^2 is squared leverage, and ε is an error term. The inclusion of squared leverage is intended to pick up the non-monotonic relation between hedging and leverage. A squared term for the spot price is not included, because the univariate results in Section 5.2 indicate that there might be insufficient variation in the spot price during the sample period.

Based on the theoretical predictions, one expects the coefficient for the average spot price to be negative and the coefficients for leverage and squared leverage to be positive and negative, respectively. The comparative statics suggest a positive relation between spot price volatility and hedging delta-percentages and between market value (proxying for the relative importance of transaction costs) and hedging delta-percentages. The latter two hypotheses are based on the observation that the risk management contract initiation zone widens with an increase in volatility and tightens with an increase in transaction costs.

The regression results in Table 15 show evidence of the non-monotonic relation between leverage and hedging delta-percentages. Both leverage coefficients have the predicted signs and are significant at the 1% level. The negative relation between average spot prices and hedging delta-percentages also bears out in the regression results with a significance level of 7%. The coefficients for volatility and market value have the predicted positive signs but are not significant. The relation between volatility and hedging delta-percentages may not appear in the data because the volatility measure exhibits relatively low variation during the sample period.[26]

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[25]We reestimate the regressions described below (results available upon request) using pooled ordinary least squares adding several accounting variables previously used in the literature to explain cross-sectional variation in risk management. Data availability for the new variables leads to a significant reduction in sample size. However, the main results for average spot price, leverage, and leverage squared are unaffected.

[26]The univariate results suggest that firms with extremely low or extremely high leverage ratios may not exhibit the negative relation between spot prices and delta-percentages, which is observed for the majority

Table 15
Panel results for risk management activity

The table shows coefficient estimate (intercepts not shown), standard error, *t*-tests, *p*-values, number of observations, and adjusted fit for the following fixed-effects panel regression of firm quarters in the gold mining firm sample:

$$DP_{it} = c_i + \beta_1 Spot_t + \beta_2 Vol_t + \beta_3 MV_{it} + \beta_4 Lev_{it} + \beta_5 Lev_{it}^2 + \varepsilon_{it}, \tag{21}$$

where *DP* is the hedging delta-percentage, *Spot* is the average spot gold price (in \$/oz.) during the quarter, *Vol* is the volatility of daily gold price returns during the quarter, *MV* is the market value (in \$1,000,000) of the firm's equity at the end of the quarter, *Lev* is the firm's leverage measured as the ratio of debt to assets, Lev^2 is squared leverage, and ε is an error term. Hedging data are from the Global Gold Hedge Survey. Spot gold prices are from Datastream. All other data are from Compustat. Data are quarterly from January 1993 to March 1999. Standard errors are robust to heteroskedasticity and autocorrelation.

Variable	Estimate	Standard error	*t*-statistic	*p*-value (%)
Spot price	−0.00036	0.000197	−1.81	7.0
Volatility	0.24	0.360	0.69	49.3
Market value × 10^{-6}	9.4	9.04	1.03	30.1
Leverage	0.70	0.160	4.39	0.002
Leverage squared	−0.69	0.230	−3.01	0.3
Observations	642			
Adjusted fit	0.50			

5.4. Risk management maturity

The model also provides several predictions with respect to the maturity of the risk management instruments used. To test these predictions, the following regression model is estimated

$$Mat_{it} = c + \beta_1 Spot_t + \beta_2 MV_{it} + \beta_3 Lev_{it} + \beta_4 Lev_{it}^2 + \varepsilon_{it}, \tag{13}$$

where *Mat* is the average remaining maturity (in years) of hedging instruments weighted by delta-adjusted hedged volume and all other variables are as defined previously.[27]

The simulation results in Section 4.3.2 suggest that, within risk management price zone 2, maturity declines with the spot price. The model generates a prediction of

(*footnote continued*)
of firms with medium leverage. Thus the estimate of the spot price coefficient might be improved by accounting for leverage differences. We generate the following series of adjusted average spot prices as the product of spot price and a leverage dummy, which takes the values 0, 1, −1 for $AvgLev_i \leqslant Low$, $AvgLev_i > Low$ and $AvgLev_i < High$, and $AvgLev_i \geqslant High$, respectively. *AvgLev* is the firm's average leverage across all firm quarters, and *Low* and *High* are cut-offs that vary from 2.5% to 7.5% and from 37.5% to 42.5%, respectively. We then replace *Spot* in Eq. (12) with the adjusted series. The coefficient estimates (not shown) for the adjusted average spot price indeed tend to increase in magnitude and significance compared with the average spot price used in the base case.

[27] Firm fixed-effects are not included in this regression model because we are not aware of existing theoretical work suggesting variation in risk management maturity, which is not already contained in our set of independent variables.

Table 16
Risk management maturity
The table shows coefficient estimate (intercepts not shown), standard error, *t*-tests, *p*-values, number of observations, and adjusted fit for the following regression of firm quarters in the gold mining firm sample:

$$Mat_{it} = c + \beta_1 Spot_t + \beta_2 MV_{it} + \beta_3 Lev_{it} + \beta_4 Lev_{it}^2 + \varepsilon_{it}, \tag{22}$$

where *Mat* is the average maturity (in years) of hedging instruments weighted by delta-adjusted hedged volume, *Spot* is the average spot gold price (in \$/oz.) during the quarter, *MV* is the market value (in \$1,000,000) of the firm's equity at the end of the quarter, *Lev* is the firm's leverage measured as the ratio of debt to assets, and ε is an error term. Hedging data are from the Global Gold Hedge Survey. Spot gold prices are from Datastream. All other data are from Compustat. Data are quarterly from January 1993 to March 1999. Standard errors are robust to heteroskedasticity and autocorrelation.

Parameter	Estimate	Standard error	*t*-statistic	*p*-value (%)
Intercept	1.30	0.33	4.1	0.0001
Spot price	−0.00050	0.00091	−0.54	58.7
Market value × 10^{-6}	0.65	0.39	1.7	9.6
Leverage	4.30	0.70	6.2	0.0001
Leverage squared	−5.10	0.14	−3.8	0.0001
Observations	528			
Adjusted fit	0.18			

cross-sectional non-monotonicity: for a given spot price, firms that are far away from financial distress, firms close to but not in financial distress, and firms in financial distress should exhibit low, high, and low observed remaining maturity, respectively. Leverage is used as a proxy for the likelihood of financial distress. The comparative statics on transaction costs in Section 4.3.5 show that, given a spot price, the maturity of newly initiated contracts increases with transaction costs. However, the active risk management zone (zone 2) is wider for lower transaction costs. Thus, it could also be the case that larger firms (with lower transaction costs) adjust their contracts more frequently and thereby are more often observed with a fresh maturity and consequently exhibit longer observed maturity. Therefore, the sign of the coefficient for market value in the above regression indicates which effect might dominate.

Table 16 provides the results of the above regression. The coefficients for both leverage measures are highly significant and consistent with the predicted non-monotonic cross-sectional relation between leverage and observed maturity. The coefficient for market value has a negative sign and is significant at the 10% level, indicating that the effect of a wider active risk management zone for firms with lower transaction costs dominates their choice of shorter initial maturities. The coefficient for the average spot price is insignificant. This is most likely the result of insufficient variation in the observed spot price.

6. Conclusion

In this paper we provide a dynamic model of corporate risk management. The model analyzes the initiation, early termination, replacement, maturity and hedge ratio choice, and frequency of adjustment of risk management instruments. While static models provide valuable intuition as to why firms manage risk, we believe that this model further develops an understanding of the dynamic aspects of risk management, which could foster better understanding of observed risk management practices. The model generates many new and sometimes quite different implications as compared with related static models. Moreover, the empirical evidence suggests that the model does help explain observed risk management among gold mining companies.

We believe that the model might serve as a basis for developing normative decision tools that can assist practitioners in developing risk management strategies. For example, the model could help to develop a strategy of the optimal hedging of risks associated with predetermined long-term delivery contracts that have longer maturity than the maturity of the available risk management instruments.[28] To move closer toward developing realistic and implementable models, the current model setting could be extended along a number of dimensions. First, the current model's assumption that the firm holds no cash, which implies that it cannot use cash reserves to off-set the negative impact of distress costs, could be relaxed. Also, it is assumed that the firm has to terminate its existing risk management contract before entering into a new one. This assumption can be relaxed, if one can consider a portfolio of risk management instruments of different types. Second, one could extend the model by allowing the firm to change its debt level over time. With a dynamic capital structure, the model can be used to explore the interaction between the choices of risk management contracts and capital structure policy. The model can also incorporate potential agency problems that may arise either between managers and equityholders or between equityholders and debtholders. It appears reasonably straight-forward to modify the model to capture the extent to which the risk management dynamics can be affected by managerial compensation contracts, which arguably have shorter maturity than the firm's horizon. The model could also be extended to explore the idea of selective hedging whereby managers incorporate superior information relative to other market participants into the risk management strategy. The model results regarding the joint effects of financial distress and taxes as risk management incentives indicate that complex interactions could exist between different incentives that are difficult to capture in static models. While the above extensions could provide additional insights, they would require considerable simplification of other parts of the model and go beyond the focus of the analysis in this paper.

One could incorporate the choice of optimal hedge payoff functions in addition to the endogenous choice of the hedge ratio in the present model. Although not explicitly explored in the paper, the model implies that other features of optimal risk management design, such as the risk management contract's payoff function, could be less important than previously thought once viewed in a dynamic setting. This is because the firm can adjust its risk management contracts and thus is not

[28]For example, we can explore the case of hedging the oil-based delivery contracts entered by Metallgesellschaft described in Culp and Miller (1995) Mello and Parsons (1995).

permanently locked into a potentially suboptimal design.[29] Recognizing firms' ability to adjust risk management contracts increases the importance of the transaction costs of risk management adjustments and suggests a trade-off between optimal design and transaction costs. In such a setting it is conceivable that a firm could choose a contract whose payoff function is suboptimal (given the current state variables), but whose transaction costs of early termination are low compared with a contract with a preferable payoff function. For example, a firm might prefer an exchange-traded derivatives contract (without customization to the firm's specific situation) to an over-the-counter contract with high transaction costs of initiation and termination.

Appendix

The value of the unlevered firm $U(p)$ satisfies the equation

$$\frac{1}{2}\sigma^2 p^2 U_{pp} + (r-a)pU_p - rU + p - c - (\lambda)\max[0,\ p-c] = 0 \tag{14}$$

with additional boundary condition $U(p) > 0$ corresponding to the case in which the firm uses its option to permanently shut down its operations, if the spot price drops far below its production costs c.

We now describe the numerical algorithm that is applied to solve stochastic control problems Eqs. (5) and (6). For each case, one needs to find a solution that satisfies simultaneously the maximization problems and partial differential equations. The algorithm is based on the finite-difference method augmented by a policy iteration.[30]

The calculations are complicated by the fact that these are infinite horizon stochastic optimization problems, in which the values of the equity and debt are time-independent. Therefore, numerical solutions require reformulating the model into a finite horizon approximation.[31] The procedure is initialized by approximating (guessing) values for the functions in each node of the terminal time period. This reformulation effectively implies that a derivative with respect to time is added to the equations of each optimal stochastic control problem. For example, in the valuation problem for the all-equity firm, a new term E_t is added to the left-hand side in Eq. (5). The errors that result from the approximation of functions at the terminal time can be reduced by increasing the length of the horizon of the problem and iterating until the derivative E_t is indistinguishable from zero for each node on the grid.

[29] Brown and Toft (2002) provide results as to how quantity risk and variation in the risk management horizon affect optimal hedge ratios and payoff functions, which are also studied by Adam (2002a). The effect of quantity risk on optimal hedging is also addressed by Banerjee and Noe (2001).

[30] See, for example, Kushner and Dupuis (1992), Barraquand and Martineau (1995), and Langetieg (1986) for the theory and applications of numerical methods for solving stochastic control problems.

[31] Flam and Wets (1987), and Mercenier and Michel (1994), discuss the approximation of infinite horizon problems in deterministic dynamic programming models.

We use a discrete grid and a discrete time step $\Delta t = \Delta\tau$. The state space (p, p^*, h, τ) is discretized using a three-dimensional grid $N_p \times N_{p*} \times I \times N_\tau$ with corresponding spacing between nodes in each dimension of $\Delta p, \Delta p^*$, Δh and $\Delta\tau$ and where $\Delta X = (X_{\max} - X_{\min})/N_x$ and $X \in \{p, p^*, h, \tau\}$; $X_{\max}$ and $X_{\min}$ are the upper and lower boundaries. The grid sizes in dimension of p, p^* and τ are chosen to achieve stability of the algorithm. In each node on the grid (p, p^*, h, τ) the partial derivatives are computed according to the Euler method. For example, the first and second derivatives of the equity value with respect to p and p^* are

$$\mathrm{E}_p(p, p^*, h, \tau) = \frac{\mathrm{E}(p + \Delta p, p^*, h, \tau) - \mathrm{E}(p - \Delta p, p^*, h, \tau)}{2\Delta p}, \tag{15}$$

and

$$\mathrm{E}_{pp}(p, p^*, h, \tau) = \frac{\mathrm{E}(p + \Delta p, p^*, h, \tau) - 2\mathrm{E}(p, p^*, h, \tau) + \mathrm{E}(p - \Delta p, p^*, h, \tau)}{\Delta p \Delta p}, \tag{16}$$

with appropriate modifications at the grid boundaries.

Initially one needs to calculate the value of the unlevered firm $U(p)$, which depends upon the price level only. Its value satisfies the PDE in Eq. (14) which can be numerically solved using a standard explicit finite-difference scheme taking into account the boundary condition, $U(p) \geqslant 0$.

The procedure for the calculation of the equity and debt value is more complex because it incorporates the decision to initiate the risk management contract. The procedure starts with the approximation of the values of the equity and debt for the terminal time t. In each node (p, p^*, h, τ) the terminal values are set $\mathrm{E}_{(t)}(p, p^*, h, \tau) = \max(0,\ U(p) - d/r) + h\,V(p, p^*, h, \tau)$ and debt $D_{(t)}(p, p^*, h, \tau) = (1 - DC)U(p)$ if $\mathrm{E}_{(t)}(p, p^*, h, \tau) = 0$, otherwise $D_{(t)}(p, p^*, h, \tau) = d/r$. This approximation does not match the exact values of the equity and debt because default option and hedging option are ignored. However, by running backward recursion for a relatively large number of Δt steps, the values on the grid converge to the steady state values because the initial misspecifications of the terminal values are smoothed away due to discounting. Thus, working backward in time for each node on the grid according to the explicit finite-difference scheme and taking into account the risk management decision, the value of equity $\mathrm{E}_{(t-\Delta t)}$ at each node (p, p^*, h, τ) at time $t - \Delta t$ is determined as

$$\begin{aligned}
&\mathrm{E}_{(t-\Delta t)}(p, p^*, h, \tau) \\
&\quad = \max_{i \in \{\{h,\tau\},0,-1\}} \Bigg[\Bigg(p' - c - d - (\lambda) \max[0,\ p' - c - d] \\
&\qquad - C^P_{\text{Distress}}\, \delta \left[\frac{D_{(t)}}{\mathrm{E}_{(t)} + D_{(t)}} - L \right] \max[0,\ \ -p' + c + d] \Bigg) \Delta t + \mathrm{e}^{-r\Delta t} \mathbb{E}_Q[\mathrm{E}_{(t)}] \Bigg] \\
&\quad = \max_{i \in \{\{h,\tau\},0,-1\}} \Bigg[\Bigg(p' - c - d - (\lambda) \max[0,\ p' - c - d]
\end{aligned}$$

$$- C^P_{\text{Distress}}\,\delta\left[\frac{D_{(t)}}{\mathrm{E}_{(t)}+D_{(t)}} - L\right]\cdot \max[0,\ -p' + c + d]\Big)\Delta t$$

$$+ \mathrm{E}_{(t)}(p,p^*,h,\tau) + \Delta t \mathscr{L}[\mathrm{E}_{(t)}(p,p^*,h,\max(0,\tau-\Delta t))]\Bigg], \tag{17}$$

where $\mathscr{L}[\mathrm{E}^U_{(t)}(p,p^*,h,\max(0,\tau-\Delta t))]$ is the differential operator applied to $\mathrm{E}_{(t)}$ in node $(p,p^*,h,\max(0,\tau-\Delta t))$

$$\mathscr{L}[Z] = \frac{1}{2}\,\sigma^2 p^2 Z_{pp} + (r-a)pZ_p + (r-a)p^* Z_{p^*} - Z_\tau - rZ, \tag{18}$$

in which all partial derivatives are computed according to the Euler method, where $\max(0,\tau-\Delta t)$ is the remaining maturity of the contract (if any) at time t given the maturity τ at time $t-\Delta t$. In (17) $p' = p$, if $\tau = 0$, or $p' = hp^* + (1-h)p$ otherwise, where p^* is the price guaranteed by the risk management contract originated earlier. The second equality in Eq. (17) comes from the Euler decomposition of Eqs. (5) and (6) in which a new term E^U_t is added, where $\mathrm{E}^U_t = \mathrm{E}^U_{(t+\Delta t)} - \mathrm{E}^U_{(t)}/\Delta t$.

If $\tau = 0$, we check whether or not it is optimal for the equityholders to initiate a risk management contract. The equityholders initiate the contract, if, for some $\{h,\tau\}$,

$$\mathrm{E}_{(t)}(p,p^*,h,0) < \max_{h\subseteq\{h_1,h_2\ldots h_I\},\tau\leqslant T}[\mathrm{E}_{(t)}(p,p,h,\tau) - TC], \tag{19}$$

where TC are transaction costs. If inequality Eq. (19) is satisfied, then the equity value is set equal to the maximum over all admissible h and τ in the right-hand side of Eq. (19).

Also, at each node we check whether or not the equity value (without value of the contract) $\mathrm{E}_{(t)}(p,p^*,h,\tau)$ is non-negative. If the equity value becomes negative, default occurs, i.e., if

$$\mathrm{E}_{(t)}(p,p^*,h,\tau) < h\,V(p,p^*,\tau), \tag{20}$$

then the equityholders default.

We repeat this backward induction procedure for $t-2\Delta t,\ t-3\Delta t,\ldots,t-N\Delta t$ until the value function for $\mathrm{E}_{(t)}(p,p^*,h,\tau)$ reaches a steady state in each node on the grid, i.e., until $\max_{(p,p^*,h,\tau)}|\mathrm{E}_{(t)}(p,p^*,h,\tau)\ - \mathrm{E}_{(t-\Delta t)}(p,p^*,h,\tau)| < \varepsilon$, where ε is the predetermined accuracy level. We find this procedure to be robust to the choice of the values at the terminal time. The procedure for the computation of the debt value is similar.

References

Adam, T., 2002a. Risk management and the credit risk premium. Journal of Banking and Finance 26, 243–269.

Adam, T., 2002b. Why firms use non-linear hedging strategies. Unpublished working paper, Hong Kong University of Science and Technology.

Allayannis, G., Brown, G.W., Klapper, L.F., 2003. Exchange rate risk management: evidence from East Asia. Journal of Finance 58, 2667–2710.

Altman, E., 1968. Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. Journal of Finance 23, 589–609.

Banerjee, S., Noe, T.H., 2001. Exotics and electrons: electric power crises and financial risk management. Unpublished working paper, Tulane University, New Orleans, Louisiana.

Barraquand, J., Martineau, D., 1995. Numerical valuation of high dimensional multivariate American securities. Journal of Financial and Quantitative Analysis 30, 383–405.

Brown, G.W., Toft, K.B., 2002. How firms should hedge. Review of Financial Studies 15, 1283–1324.

Brown, G.W., Crabb, P.R., Haushalter, D., 2001. Are firms successful at selective hedging? Unpublished working paper, University of North Carolina, Chapel Hill, North Carolina.

Cooper, I., Mello, A., 1999. Corporate hedging: the relevance of contract specifications and banking relationships. European Finance Review 2, 195–223.

Culp, C., Miller, M., 1995. Metallgesellschaft and the economics of synthetic storage. Journal of Applied Corporate Finance 7, 62–75.

DeMarzo, P., Duffie, D., 1995. Corporate incentives for hedging and hedge accounting. Review of Financial Studies 8, 743–771.

Dolde, W., 1993. The trajectory of corporate financial risk management. Journal of Applied Corporate Finance 6, 33–41.

Fama, E., French, K., 1988. Business cycles and the behavior of metals prices. Journal of Finance 43, 1075–1093.

Ferguson, M.F., Mann, S.C., 2001. Execution costs and their intraday variation in futures markets. Journal of Business 74, 125–160.

Flam, S., Wets, R.J.-B., 1987. Existence results and finite horizon approximates for infinite horizon optimization problems. Econometrica 55, 1187–1209.

Froot, K., Scharfstein, D., Stein, J., 1993. Risk management: coordinating corporate investment and financing policies. Journal of Finance 48, 1629–1658.

Géczy, C., Minton, B.A., Schrand, C., 1997. Why firms use currency derivatives. Journal of Finance 52, 1323–1354.

Géczy, C., Minton, B.A., Schrand, C., 1999. Choices among alternative risk management strategies: evidence from the natural gas industry. Unpublished working paper, University of Pennsylvania Wharton School, Philadelphia, Pennsylvania.

Gibson, R., Schwartz, E., 1990. Stochastic convenience yield and the pricing of oil contingent claims. Journal of Finance 45, 959–976.

Ho, T.S.Y., 1984. Intertemporal commodity futures hedging and the production decision. Journal of Finance 39, 351–376.

Huang, R., Masulis, R., 1999. FX spreads and dealer competition across the 24 hour day. Review of Financial Studies 12, 61–93.

Jensen, M., Meckling, W., 1976. Theory of the firm: managerial behavior, agency costs, and ownership structure. Journal of Financial Economics 4, 305–360.

Kerkvliet, J., Moffett, M., 1991. The hedging of an uncertain future foreign currency cash flow. Journal of Financial and Quantitative Analysis 26, 565–578.

Koppenhaver, G., 1985. Bank funding risks, risk aversion, and the choice of futures hedging instrument. Journal of Finance 40, 241–255.

Kushner, H., Dupuis, P., 1992. Numerical Methods for Stochastic Control Problems in Continuous Time. Springer, Germany.

Langetieg, T., 1986. Stochastic control of corporate investment when output affects future prices. Journal of Financial and Quantitative Analysis 21, 239–263.

Leland, H., 1998. Agency costs, risk management, and capital structure. Journal of Finance 53, 1213–1243.

Mauer, D., Triantis, A., 1994. Interactions of corporate financing and investment decisions: a dynamic framework. Journal of Finance 49, 1253–1277.

Mello, A., Parsons, J.E., 1995. Maturity structure of a hedge matters: lessons from the Metallgesellschaft debacle. Journal of Applied Corporate Finance 8, 106–120.

Mercenier, J., Michel, P., 1994. Discrete-time finite horizon approximation of infinite horizon optimization problems with steady-state invariance. Econometrica 62, 635–656.

Mian, S.L., 1996. Evidence on corporate hedging policy. Journal of Financial and Quantitative Analysis 31, 419–439.
Morgan, G.E., Shome, D., Smith, S.D., 1988. Optimal futures positions for large banking firms. Journal of Finance 43, 175–195.
Naik, N.Y., Yadav, P., 2003. Risk management with derivatives by dealers and market quality in government bond markets. Journal of Finance 58, 325–353.
Nance, D., Smith, C.W., Smithson, C., 1993. On the determinants of corporate hedging. Journal of Finance 48, 267–284.
Opler, T., Titman, S., 1994. Financial distress and corporate performance. Journal of Finance 49, 1015–1040.
Petersen, M.A., Thiagarajan, S.R., 2000. Risk measurement and hedging: with and without derivatives. Financial Management 29, 5–30.
Schwartz, E., 1997. The stochastic behavior of commodity prices: implications for valuation and hedging. Journal of Finance 52, 923–973.
Simkins, B.J., Carter, D.A., Rogers, D.A., 2004. Does fuel hedging make economic sense? The case of the US airline industry. Unpublished working paper, Oklahoma State University, Stillwater, Oklahoma.
Smith, C.W., Stulz, R.M., 1985. The determinants of firms 'hedging policies'. Journal of Financial and Quantitative Analysis 20, 391–405.
Stulz, R.M., 1984. Optimal hedging policies. Journal of Financial and Quantitative Analysis 19, 127–139.
Stulz, R.M., 1996. Rethinking risk management. Journal of Applied Corporate Finance 9, 8–24.
Stulz, R.M., 2002. Risk Management and Derivatives. South-Western College Publishing, Mason, Ohio.
Titman, S., Tsyplakov, S., 2002. A dynamic model of optimal capital structure. Unpublished working paper, University of South Carolina, Columbia, South Carolina.
Tufano, P., 1996. Who manages risk? An empirical examination of risk management practices in the gold mining industry. Journal of Finance 51, 1097–1137.