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Fair Premium Rating Methods and the Relations Between Them

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ABSTRACT

This article considers several well known methods of calculating fair premium rates. Particular reference is made to the Myers-Cohn and internal rate of return methods. In the absence of taxes, the most natural application of each method produces a return on equity, period by period, which is consistent with the capital asset pricing model. Hence, the two methods produce the same premium under these conditions. The same result holds in the presence of tax, but only with the addition of further conditions relating to the insurer's capital structure and rate of release of underwriting profits.

Introduction

The operation of an insurance business requires capital to provide security that claims will be met. This article is concerned with the case in which this capital is provided by shareholders (as opposed to the case of a mutual insurer). Shareholders' capital is placed at risk by the insurance process and so must attract a return commensurate with the risk to which it is subject.

To the extent that the return exceeds that which would be generated by the mere holding of the capital base in investments, the excess must be drawn from premiums paid by policyholders. Hence, these premiums must be based on a pricing formula of the form

$$\text{premium} = \text{risk premium} + \text{expense loading} + \text{margin to service capital.} \quad (1)$$

The margin appearing as the final premium component in equation (1) will be referred to as the profit margin.

Fair premium rating methods are those which quantify the profit margin in such a way as to provide shareholders with a fair return on capital but no

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more. Details differ from one method to another, but all have this concept of fairness at their base.

Several fair rating methods have been developed, and the major ones are summarized below. The methods yield different results when applied to a specific numerical case, even though each has been developed more or less rigorously from a certain theoretical base. Thus, the methods are not equivalent in general. Nevertheless, their derivations indicate that they have a good deal in common. It is therefore useful to render manifest those equivalences which do exist by identifying circumstances in which different fair rating methods yield identical results. The obverse of the establishment of these equivalences will be the identification of the major differences between methods. The identification of these equivalences and differences is the purpose of this article, and, in this respect, it forms a sequel to Cummins (1990), which compared the Myers and Cohn and internal rate of return methods of rating.

Notation, Terminology, and Conventions

Consider a portfolio of insurance contracts. The parties financially involved in this portfolio comprise policyholders, shareholders, issuers of investments to the insurer, tax authorities, and providers of goods and services to the insurer. To anticipate the more detailed treatment given below, the cash flows between these parties comprise premiums, claims and expenses, investment income, taxes, and payments to and from shareholders. Subsequent sections will consider the subset of this portfolio consisting of policies underwritten in a particular period. This subset will be referred to as the cohort. It will be traced from its origins in the underwriting period, through subsequent periods, until it generates no further transactions.

Let the various periods be denoted by $t = 0, 1, ..., T$, with $t = 0$ denoting the underwriting period, and $t = T$ the period in which the final transaction occurs. All periods t are of equal length, but the length is unspecified; it may be years, quarters, months, etc. The premium underwritten in period zero provides coverage during periods 0, 1, ..., k, where k is a fixed but arbitrary integer less than or equal to T. The $T + 1$ periods will span a time interval of $T + 1$ units. By convention, this interval will be $[0, T+1]$. That is, period t corresponds to the interval $[t, t+1]$.

A convention is adopted whereby premiums payable in period t are assumed payable at the commencement of that period, whereas all cash flows are assumed payable at the end of the period. There may also be a capital flow at the commencement of period zero. Any up-front expenses, which would be paid at the time of premium receipt (commencement of period), could be accommodated by regarding "premiums" as in fact premiums net of concurrent expenses. This convention, while unrealistic, greatly simplifies the algebra without any essential change to the conceptual situation. In principle, it can be made to approximate the realistic situation arbitrarily closely by reducing the length of the periods t (and increasing T) indefinitely.

In the following notation, a subscript t (= 0, 1, ..., T) indicates the epoch (rather than the period) to which the cash flow designated by the primary symbol relates. A symbol with a tilde over it is a random variable. The same symbol without the tilde represents the expected value of that variable; for example, $E[\tilde{C}_t] = C_t$. The same symbol with an overbar represents the centralized version of the random variable; for example, $\bar{C}_t = \tilde{C}_t - C_t$.

Let P_t = gross premium,

$\tilde{E}_t$ = expenses associated with the administration of claims,

$\tilde{C}_t$ = claim payments,

$\tilde{I}_t$ = investment income,

$\tilde{T}_t$ = tax payments,

$\tilde{F}_t$ = net cash flow to shareholders,

A_t = amount of assets under management by the insurer in respect of the cohort,

$\tilde{Q}_t$ = insurance profit,

K_t = insurer's capital,

R_t = technical reserves, consisting of the sum of loss reserve and unearned premium reserve,

τ_t = tax rate,

$\tilde{i}_{At}$ = rate of return on A_t,

i_{Ft} = risk-free rate of return, and

$\tilde{i}_{Mt}$ = share market rate of return.

Tax payments $\tilde{T}_t$ consist of the two components $\tilde{T}_t^{(u)}$ = tax on underwriting profit, and $\tilde{T}_t^{(i)}$ = tax on investment income.

In this notation, all balance sheet items carrying the subscript t, such as A_t, K_t, etc., denote the values of those items at the end of period t-1, that is, after any cash flows during that period or, equivalently, at the beginning of period t. However, R_0 is in principle allowed to be nonzero. Effectively, this permits any margin of profit or loss in premium, as measured in relation to technical reserve, to be recognized at time zero precisely by release from (or absorption by) technical reserves. In fact, in the specific cases addressed in this article, R_0 will be zero. Moreover, K_0 is not necessarily zero. This allows the possibility of capital contribution at the beginning of the underwriting period. For any rate of return $\tilde{i}_{Xt}$, X = A, F, M, etc., $\tilde{r}_{Xt}$ is defined by

$$\tilde{r}_{Xt} = 1 + \tilde{i}_{Xt}. \tag{2}$$

All rates of return are random variables, assumed subject to the capital asset pricing model (CAPM) (Lintner, 1965; Mossin, 1966; Sharpe, 1964):

$$\tilde{r}_{Xt} = r_{Ft} + \beta_X [r_{Mt}-r_{Ft}] + \varepsilon_{Xt}, \tag{3}$$

$$\text{where } E[\varepsilon_{Xt}] = 0, \text{ and} \tag{4}$$

$$\beta_X = Cov[\tilde{r}_{Xt}, \tilde{r}_{Mt}]/V[\tilde{r}_{Mt}], \tag{5}$$

which is the beta, or volatility, of rate of return $\tilde{r}_{Xt}$.

It is convenient to formulate rates of return in terms of the CAPM since this is the basis of all the models discussed here. However, it may be noted also that nowhere does the reasoning of this article call on the specific form of equation (5). All that is required of β_X is the weaker linearity property that $\beta_X = a_1\beta_{X_1} + a_2\,\beta_{X_2}$ for $X = a_1\,X_1 + a_2\,X_2$ with a_1 and a_2 constants. Any relations demonstrated in later sections to hold between the models described in the next section would continue to hold if those models were extended from a CAPM framework to one of linear factor asset pricing models.

Define

$$\tilde{r}_{Xs:t} = \tilde{r}_{Xs}\,\tilde{r}_{Xs+1}\,\ldots\,\tilde{r}_{Xt} \text{ for } t \geq s. \tag{6}$$

= accumulation factor from beginning of period s to end of period t, that is, from epoch s to epoch t + 1, and define $r_{Xs:t}$ similarly. Note that $\tilde{r}_{Xt:t} = \tilde{r}_{Xt}$. By convention, $\tilde{r}_{Xt:t-1} = 1$.

The naive generalization from single-period to multiperiod CAPM embodied in equation (6) will in fact be subject to some conditions on the process which generates the fluctuation in r_F and r_M from period to period (see, e.g., Constantinides, 1978). These technical issues are not discussed here, particularly as the main results obtained below require constant rates of return from period to period.

It will be necessary to make use of a particular result referred to here as Myers' theorem. It is quoted and proven in the Appendix and, for convenience, is restated below. An earlier proof appeared in Derrig (1985).

Myers' Theorem

Consider an asset of unit value at time zero. Suppose this asset generates a (stochastic) return of $\tilde{r}_s$ in period s = 0, 1, ...t, and that tax on this return becomes payable at rate τ immediately at the end of period s. After-tax returns are assumed reinvested in the same asset. The present value of the expected tax payable at the end of period t can be calculated as if returns are risk free throughout. That is, the required present value is $\tau i_{Ft} r_{F0:t-1}/r_{F0:t}$.■

Only a single tax rate is contemplated above. In practice, different components of profit may be subject to different tax rates. Again, accommodation of this feature in the discussion would not be especially difficult but would considerably clutter the algebra without changing any of the essential ideas.

Fair Premium Rating Methods

General

The basic characteristics of fair premium rating methods were described above. A number of such methods have been used widely in the past. They are described chronologically in the following four subsections. It is not my intention to provide the theoretical groundwork for these methods, which may be obtained from the references given. Thus, only a minimal description of each method is given. Cummins and Harrington (1987) provide a useful collection of a number of the most influential references. The first two methods, those of

Fairley and Hill and Modigliani, express the fair profit margin in terms of a single equation involving global parameters of the insurance portfolio. The remaining two methods express this margin as the implicit solution of a cash flow equation involving all the detail of cash flows generated by the cohort.

Fairley Method

The Fairley (1979) method deals with the situation in which all premiums are received, and associated expenses paid, at the beginning of the period of coverage:

$$P_0 = P,$$
$$P_t = 0,\ t = 1, 2, ...,T. \tag{7}$$

Then gross premiums may be related to claims and expenses through a (proportionate) profit margin η:

$$P - (C+E) = \eta P, \tag{8}$$

where

$$C = \sum_{t=1}^{T} C_t$$

= total expected claims cost, undiscounted for investment income, (9)
and

$$E = \sum_{t=1}^{T} E_t$$

= total expected claims administration expenses. (10)
Direct loss adjustment expenses may be included in C_t or E_f. In practice, it is common to find them included in C_t.

Fairley defines two global parameters:

k = funds generating coefficient
= ratio of loss reserves to premium;
S = premium to surplus ratio.

Both definitions lack precision. Below, I give more precise definitions which are consistent with the Fairley formula:

$$\eta = -ki_L + \tau i_F/S(1-\tau), \tag{11}$$

where $1 + i_L = r_L = r_F + \beta_L(r_M - r_F)$ (cf. equation [3]), (12)

with β_L the insurance liabilities beta, defined by equation (5) with $\tilde{r}_L$ representing the (stochastic) factor by which a particular set of claims liabilities changes over a period. Note that the Fairley model is a single-period model; hence, the subscript t is omitted as redundant.

The liabilities beta is expressed by Fairley (see (1.6a) of Cummins and Harrington, 1987) as

$$\beta_L = [(kS+1)\beta_A - \beta_E/(1-\tau)]/kS, \tag{13}$$

where β_A = volatility of insurer's asset portfolio, and
β_E = volatility of insurer's equity.

Hill-Modigliani Model

Hill and Modigliani (1981) were motivated by two main considerations: (1) U.S. tax rates differed between different types of assets; and (2) insurer asset portfolios generally contained significant proportions of nontraded assets, and these were thought to be subject to different betas from those applicable to traded assets.

Suppose investment income is generated by a portfolio of assets consisting of proportions ξ_1, ξ_2, etc. (by market value) invested in assets with expected rates of return i_1,i_2, etc. and subject to tax rates τ_1, τ_2, etc. Then the average tax rate on investment income is

$$\tau = \sum_j \xi_j i_j \tau_j / \sum_j \xi_j i_j. \tag{14}$$

Suppose that assets A comprise traded stocks T and nontraded stocks N: A = T + N. Suppose that the traded and nontraded stocks are subject to betas of β_T and β_N, respectively. Also define k_N to equal an analogue of funds generating coefficient for nontraded stocks. Then Hill and Modigliani give the following result (see Cummins and Harrington, 1987, equation (2.13)) in place of equation (13):

$$\beta_L = [(kS+1)\beta_A + k_N S\beta_N - \beta_E/(1-\tau)]/[(k+k_N)S], \tag{15}$$

from which η is calculated by means of equations (11) and (12) as before. The definition of k_N lacks precision in the same way as did k in the Fairley method. Again, the necessary precision will be added in a later section.

Myers-Cohn Method

The Myers and Cohn (1981) model is a multiperiod model, tracing all expected cash flows of the cohort. In its bare essentials, the method consists of determining premium as the quantity that equates the present values of certain cash flows, as follows:

Present value of premiums = present value of claim payments
\+ present value of expenses
\+ present value of tax on profit derived from underwriting (including investment income generated by loss reserves)
\+ present value of tax on investment income generated by surplus funds provided by shareholders. (16)

The rates of discount to be used in the computation of the present values differ from one term to another, as follows.

Cash Flow	Discount Rate
Premiums	Risk Free
Acquisition Expenses	Risk Free
Claims	Based on β_L
Claims Administration Expenses	Based on β_L
Tax on Underwriting Profit	Based on β_L
Tax on Investment Income	Risk Free

Thus, equation (16) may be written in the form

$$V(P, r_F) = V(C+E, r_L) + V(T^{(u)}, r_F, r_L) + V(T^{(i)}, r_F), \tag{17}$$

where

$$V(Z, r_X) = \sum_{t=0}^{T} r_{X0:t}^{-1} Z_t. \tag{18}$$

The term $V(T^{(u)}, r_F, r_L)$ involves two rates of return since underwriting profit of a period (and therefore tax on it) consists of a difference between premiums earned and claims incurred. The first of these components is discounted at r_F, the second at r_L.

This definition of the Myers-Cohn method slightly generalizes the original, which assumed constant rates of return from period to period. As noted above, such a generalization will be valid only under certain technical conditions. The main results obtained below in relation to the Myers-Cohn method apply to the less general situation considered by Myers and Cohn.

Note that in the term $V(T^{(i)},r_F)$ of equation (17), the profits on which tax is based are calculated on the basis of risk-free rates of return and using technical reserves, which discount the relevant cash flows at risk-free rates of return.

Note also that, if $P_t \neq 0$ for more than one value of t, then equation (17) will not be sufficient to define all P_t. Further constraints will be required. These could take various forms, but it is convenient to make the most common assumption here, namely that the distribution of premium by period is given, and only its total quantum needs to be fixed by equation (17). That is, $P_t = P\phi_t$, with

$$\sum_{t=0}^{T} \phi_t = 1,$$

the ϕ_t being given and P unknown.

The Internal Rate of Return Method

The internal rate of return model, described by Cummins (1990), is another multiperiod model. Like the Myers-Cohn model, it traces all expected cash flows of the cohort. It consists of determining premium as the quantity which sets to zero the net present value of all payments to and from shareholders, evaluated at a suitable rate of return on equity. That is, premium must satisfy

$$\sum_{t=0}^{T} r_{I0:t}^{-1} F_t = 0, \tag{19}$$

where $i_I = r_I - 1$ is the return on equity judged appropriate for an insurer with the capital structure consistent with that which would emerge from continued underwriting of the line of insurance under consideration. Any imprecision in this method lies in the definition of r_I, and this will be discussed below.

Cash Flows of an Insurance Operation

The various types of cash flow involved in an insurance operation that were listed above are illustrated schematically in Figure 1. Of the six types of cash flows illustrated, three may be regarded as inputs to the system, namely premiums, claims and expenses. Of the other three types, investment income and tax are derived quantities, dependent on the inputs. Flows to shareholders may be considered partly inputs (capital) and partly derived (revenue returned on that capital).

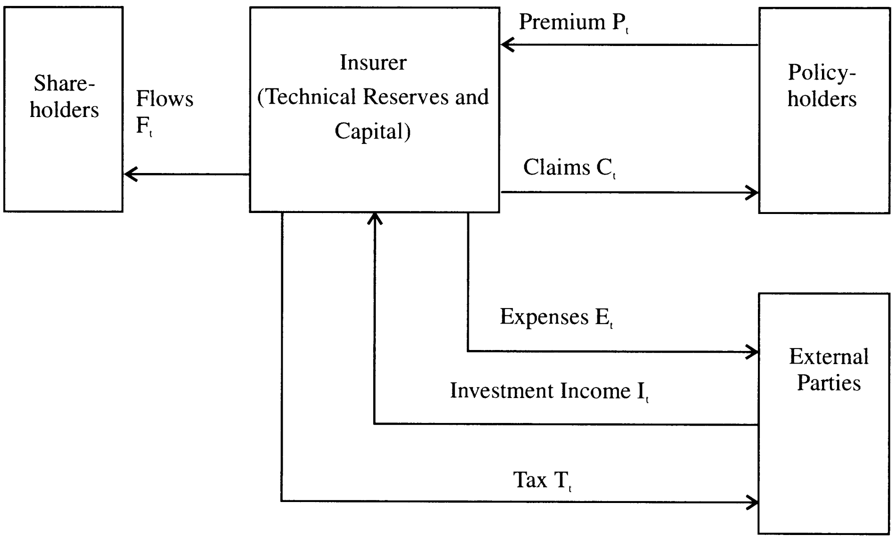

Figure 1
Cash Flows Generated by Insurance Underwriting

It is possible to set out the relation between each of the derived quantities and the inputs that determine it. The most basic of all is the relation describing the closure of the illustrated system:

$$P_t - \tilde{C}_{t+1} - \tilde{E}_{t+1} + \tilde{I}_{t+1} - \tilde{T}_{t+1} - \tilde{F}_{t+1} = (R_{t+1}+K_{t+1}) - (R_t+K_t) \text{ for each t.} \tag{20}$$

In addition,

$$\tilde{I}_{t+1} = \tilde{i}_{At}(A_t+P_t), \text{ and} \tag{21}$$

$$T_t = \tau_t \tilde{Q}_t. \quad (22)$$

The quantities on the right-hand side of equations (21) and (22) are themselves derived quantities as follows:

$$A_t = R_t + K_t, \text{ and} \quad (23)$$

$$R_t = \sum_{s=t}^{T} [r_{Lt:s}^{-1}(1+v_{s+1})(C_{s+1}+E_{s+1})-r_{Ft:s-1}^{-1}P_s], \; t = 0,...,T, \quad (24)$$

where v_s is a loading factor applied to expected costs of period s in their contribution to loss reserves.

Equation (24) defines loss reserves in terms of discounted expected claim payments. This conflicts with accepted practice in some countries, such as the United States and the United Kingdom, where traditionally reserving has been carried out on an undiscounted basis. However, equation (24) will be central to the conditions under which equivalence between the Myers-Cohn and internal rate of return methods will be established.

The variation of the factor v with period is a little more general than is usually considered. For example, Myers and Cohn (1981, p. 68) assume that "underwriting profits are accrued as losses are paid." Effectively, this amounts (apart from the fact that the Myers-Cohn method is described in that work in terms of undiscounted loss reserves) to assuming $v_s = v$, independent of s, in equation (24) with υ such that $R_0 = 0$.

Now,

$$\tilde{Q}_t = P_{t-1} + \tilde{I}_t - \tilde{E}_t - \tilde{C}_t - (R_t - R_{t-1}). \quad (25)$$

At this stage the capital quantities K_t may be regarded as inputs. It is convenient to represent equation (24) in the form

$$R_t = R_t^C - R_t^P, \quad (26)$$

where R_t^C, R_t^P denote the two separate summations appearing on the right side of equation (24) for t = 0, ..., T.

Note that, in the case t = 0, equation (24) yields

$$R_0 = R_0^C - R_0^P$$
$$= \sum_{s=0}^{T} r_{L0:s}^{-1}(1+v_{s+1})(C_{s+1}+E_{s+1})-\sum_{s=0}^{T} r_{F0:s-1}^{-1}P_s. \quad (27)$$

As remarked above, this need not be equal to zero in general, since, for given C_s and E_s, the v_s and P_s may be adjusted at will. It is useful to note, however, that equation (17) is equivalent to $R_0 = 0$ in the absence of taxes ($T_t = 0$) and loading factors ($v_t = 0$); that is, $R_0 = 0$ for Myers-Cohn premiums in the absence of taxes and loading factors.

Note also that equation (24) enables R_{t+1} and R_t to be related. For

$$r_{Lt} R_t^C = (1+v_{t+1}) (C_{t+1}+E_{t+1}) + R_{t+1}^C, \quad (28)$$

$$r_{Ft} R_t^P = r_{Ft} P_t + R_{t+1}^P; \quad (29)$$

hence, for any X,

$$r_{Xt} R_t - R_{t+1} = (r_{Xt}-r_{Lt}) R_t^C -(r_{Xt}-r_{Ft}) R_t^P + (1+v_{t+1}) (C_{t+1}+E_{t+1}) - r_{Ft} P_t. \quad (30)$$

Consider now the flows F_{t+1} to shareholders which, by equation (20), may be put in the form

$$\begin{aligned}\tilde{F}_{t+1} &= (K_t-K_{t+1}) + (R_t-R_{t+1}) + P_t - \tilde{C}_{t+1} - \tilde{E}_{t+1} + \tilde{I}_{t+1} - \tilde{T}_{t+1}.\\ &= (K_t-K_{t+1}) + \tilde{Q}_{t+1} - \tilde{T}_{t+1} \text{ (by equation [25])}\\ &= (K_t-K_{t+1}) + (1-\tau_{t+1}) \tilde{Q}_{t+1} \text{ (by equation [22]).} \quad (31)\end{aligned}$$

By equations (21), (23), and (25), the term $\tilde{Q}_{t+1}$ in equation (31) may be expressed as

$$\begin{aligned}\tilde{Q}_{t+1} &= P_t + \tilde{i}_{At} (R_t+K_t+P_t) - \tilde{E}_{t+1} - \tilde{C}_{t+1} + (R_t-R_{t+1})\\ &= i_{At} K_t +(r_{At} R_t-R_{t+1})+ r_{At} P_t- \tilde{E}_{t+1}- \tilde{C}_{t+1} + \bar{i}_{At}(R_t+K_t+P_t)\\ &= i_{At} K_t + (i_{At}-i_{Lt}) R_t^C - (i_{At}-i_{Ft})(R_t^P-P_t)\\ &\quad + v_{t+1}(C_{t+1}+E_{t+1}) - (\bar{C}_{t+1}+\bar{E}_{t+1})+ \bar{i}_{At} (R_t+K_t+P_t), \quad (32)\end{aligned}$$

by equation (30).

Finally, equation (32) enables equation (31) to be written as

$$\tilde{F}_{t+1} = \underbrace{(K_t-K_{t+1})}_{\text{Return of Capital}} + \underbrace{K_t(1-\tau_{t+1})[i_{At}+(i_{At}-i_{Lt}) R_t^C/K_t+v_{t+1}(C_{t+1}+E_{t+1})/K_t-(i_{At}-i_{Ft})(R_t^P-P_t)/K_t+\tilde{z}_{t+1}]}_{\text{Income to Shareholders}}, \quad (33)$$

where $\tilde{z}_{t+1} = [-(\bar{C}_{t+1}+\bar{E}_{t+1}) + \bar{i}_{At} (R_t+K_t+P_t)]/K_t$, which is a stochastic variable with zero mean. This equation holds for all t = 0, 1, ..., T.

Note that, since K_t is the amount remaining during period t of the capital contributed by shareholders, the rest of the income term in equation (33) is the return on equity paid on capital in that period; that is,

$$ROE_t = (1-\tau_{t+1}) \left\{ i_{At}+(i_{At}-i_{Lt})\theta_t^{-1}\left[1-\frac{\beta_A}{\beta_A-\beta_L}\frac{R_t^P-P_t}{R_t^C}\right]+v_{t+1}(C_{t+1}+E_{t+1})/K_t+\tilde{z}_{t+1}\right\}, \quad (34)$$

where equation (3) has been used, and θ_t is defined by

$$\theta_t = K_t/R_t^C, \ t = 0, ..., T, \quad (35)$$

= solvency ratio relative to loss reserve.

In the usual case, P_t, R_t^P will decline quickly to zero with increasing t. If

$$R_t^P = 0 \text{ for } t = t_0, t_0+1, ..., T+1, \quad (36)$$

then equation (34) reduces to

$$ROE_t = (1-\tau_{t+1})[i_{At}+(i_{At}-i_{Lt})\theta_t^{-1} + v_{t+1}(C_{t+1}+E_{t+1})/K_t + \tilde{z}_{t+1}], \ t = t_0+1,..., T+1. \quad (37)$$

Consider the circumstances under which the cohort will provide a constant return on equity from period to period. It is best to do this in two stages, first in the absence of taxes and then with due allowance for them. In the first of these cases, equation (34) reduces to

$$ROE_t = i_{At}+(i_{At}-i_{Lt})\theta_t^{-1}\left[1-\frac{\beta_A}{\beta_A-\beta_L}\frac{R_t^P-P_t}{R_t^C}\right]+v_{t+1}(C_{t+1}+E_{t+1})/K_t+\tilde{z}_{t+1}. \quad (38)$$

Thus, the cohort will provide a constant expected return on equity from period to period if the nonstochastic part of the right side of equation (38) is constant over t; or, more specifically, if $v_t = 0$ for each t,

$$i_{At},\ i_{Lt} \text{ are independent of t;} \quad (39)$$

and

$$\theta_t = \theta\left[1-\frac{\beta_A}{\beta_A-\beta_L}\frac{R_t^P-P_t}{R_t^C}\right], \quad \theta \text{ const.} \quad (40)$$

If equation (36) holds, then equation (40) collapses to

$$\theta_t = \theta, \text{ const. for } t = t_0, ..., T. \quad (41)$$

Equation (32) identifies insurance profit as the total of income from five separate sources. First, the term involving K_t represents income generated by the assets in which shareholders' capital is held. Second, the term involving R_t^C represents income generated by assets supporting claims reserves (rate i_{At}) in excess of that anticipated by those reserves (rate i_{Lt}). Third, the term involving R_t^P represents the reduction in income generated by the premium reserve, similar in form to that from the claims reserve. Fourth, the term involving v_t represents the release to profit of loadings in loss reserves; equivalently loadings applied to the risk premium underwritten (consider the case $R_0 = 0$). Fifth, profit includes a stochastic term reflecting the uncertainty of both claims experience and investment return.

The emergence of return on equity in the form (37), or even more specifically in equation (50) below, is essentially the same as in Butsic (1991). Consider now the effect of taxes on these results. It is useful for this purpose to write the return on equity (34) in the form

$$ROE_t = ROE1_t + ROE2_t, \quad (42)$$

where

$$ROE1_t=i_{At}+(i_{At}-i_{Lt})\theta_t^{-1}\left[1-\frac{\beta_A}{\beta_A-\beta_L}\frac{R_t^P-P_t}{R_t^C}\right], \quad (43)$$

and

$$ROE2_t = (1-\tau_{t+1})\ v_{t+1}(C_{t+1}+E_{t+1})/K_t-\tau_{t+1}\ ROE1_t + (1-\tau_{t+1})\ \tilde{z}_{t+1}. \quad (44)$$

Here ROE1 is the return on equity that emerges when there are no taxes, no loading factors built into loss reserves, and no stochastic variation; ROE2 is the additional component of return on equity due to the effect of tax, loading factors in loss reserves, and stochasticity. In this case, one possibility for the cohort to provide a constant expected return on equity from period to period is that each of ROE1 and ROE2 be constant in expectation. The first of these requirements is the same as the case of no taxes, that is, is satisfied by equations (39) and (40) (or [41]). The second requirement is satisfied if

$$\tau_t \text{ is independent of t,} \tag{45}$$

and, from equation (44),

$$(1-\tau)[\nu_{t+1}/\theta_t]\frac{C_{t+1}+E_{t+1}}{R_t^C}=\tau \text{ROE1}+\text{E[ROE2]}, \tag{46}$$

where τ, ROE1, and E[ROE2] are the constant values of those quantities, and use has been made of equation (35).

Since the right side of equation (46) is a constant, this relation amounts to a requirement that ν_{t+1}/θ_t should vary inversely with the ratio $(C_{t+1}+E_{t+1})/R_t^C$. A special case of equation (46) that will prove relevant is that in which E[ROE2] = 0, and premium is all paid at time zero, so that equation (40) gives $\theta_t = \theta$ for t = 1, 2, ..., T. Then

$$\nu_{t+1}(C_{t+1}+E_{t+1})/R_t^C = [\theta\tau/(1-\tau)] \text{ ROE}, \tag{47}$$

where ROE1 has been written as just return on equity, the constant expectation of ROE_t.

Return on Equity of an Insurance Operation

Consider the capital asset pricing model return on equity. Since total liabilities during period t are equal to $R_t^C - (R_t^P-P_t)$, and capital is K_t, total assets must be

$$K_t + R_t^C - (R_t^P-P_t). \tag{48}$$

Therefore, the required return on equity is

$$\{[K_t + R_t^C - (R_t^P-P_t)]\ i_{At} - R_t^C\ i_{Lt} + (R_t^P-P_t)\ i_{Ft}\ \}/K_t, \tag{49}$$

which is seen to be equal to the nonstochastic part of the square bracketed term of equation (33), the expected return on equity actually achieved, in the absence of taxes ($\tau_t = 0$) and loss reserve loading factors ($\nu_t = 0$).

What this demonstrates is that, if technical reserves are computed according to the specifications above (see equation [24]), then

(a) In the absence of taxes and loading factors, the expected return on equity achieved in each period t = 0, 1, ..., T is precisely that required by the capital asset pricing model with due allowance for the capital structure of the insurer;

(b) This result holds independently of the level of premiums underwritten (though this will of course affect the instantaneous profit emerging at exact time zero) and of the level of capital commitment K_t;

(c) In addition to the return on equity described in (a), there may be a component of profit or loss recognized at time zero, as discussed above.

Analysis of the Internal Rate of Return Method

Suppose that all premium is paid at time zero and that the conditions of equations (39), and (40), (45), and (47) hold. By the discussion at the end of the cash flows section, the last condition implies that E[ROE2] = 0, in which case equations (42) and (43) yield a return on equity that is constant over all periods. In fact,

$$E[ROE_t] = i_A + (i_A - i_L)\, \theta^{-1}, \tag{50}$$

where i_A, i_L are the period-independent values of i_{At}, i_{Lt} in equation (39).

Now collate the following facts:

(a) By equation (33), flows $\tilde{F}_t$ to shareholders consist partly of return of capital, partly of income on outstanding capital;

(b) By equation (50), the income provides a constant expected return on equity over all periods; and hence

(c) The net present value of the expected flows F_t evaluated at the expected return on equity given by equation (50) must be zero, if there is no instantaneous profit emerging at exact time zero (see discussion in notation, terminology, and conventions section above). That is, equation (19) is satisfied when $r_{It} = r_I$, independent of t, with

$$i_I = i_A + (i_A - i_L)\theta^{-1}. \tag{51}$$

This means that the internal rate of return method will yield premiums which satisfy (c) provided that i_I is given by equation (51).

In addition, the previous section demonstrated that, in the absence of taxes, the emerging expected return on equity i_I given by equation (51) is that required in each period by the capital asset pricing model, taking into account the insurer's capital structure from one period to another.

It was noted at the end of the previous section that an additional component of profit or loss could emerge at the exact time zero. This would depend on the levels of premium and capital commitment. But as noted in (c) above, this additional component must be zero in the case that premium is evaluated by the internal rate of return method with rate of return and capitalization ratio linked by equation (51). In this case, $R_0 = 0$; if R_0 assumed any other value, that value would be recognized as profit at time zero. For $R_0 = 0$, equation (27) gives

$$\sum_{s=0}^{T} r_{F0:s-1}^{-1} \, P_s = \sum_{s=0}^{T} r_{L0:s}^{-1} \, (1+v_{s+1})(C_{s+1}+E_{s+1}). \tag{52}$$

Relation Between Myers-Cohn and Internal Rate of Return Methods

No Taxes

Consider the Myers-Cohn method in the absence of taxes. Setting $\tilde{T}_t = 0$ causes the basic Myers-Cohn equation (17) to reduce to the following:

$$V(P, r_F) = V(C+E, r_L), \tag{53}$$

where V(.,.) is defined by equation (18). This definition shows that equations (53) and (52) are equivalent if $v_s = 0$ for all s. That is to say, in the case of zero taxes, zero loss reserve loading factors, constant capitalization ratio θ, and assumption (39), the basic Myers-Cohn equation (53) (or [52]) *is derivable from the internal rate of return method.* This means that both methods must yield the same premiums.

It is to be noted that this conclusion is at variance with Cummins (1990, p. 93), who found that the Myers-Cohn and internal rate of return approaches would not necessarily give the same results (even in the present highly simplified set-up), primarily due to an effective assumption that an underwriting deficit emerges at time zero and is fully funded then by the internal rate of return method, whereas only progressively over the period of runoff by the Myers-Cohn method.

However, this underwriting deficit arises only from the undiscounted nature of the reserves assumed in the National Council on Compensation Insurance version of the internal rate of return model analyzed by Cummins (see earlier comments on this above). In the present context, loss reserves are given by equation (24) with v = 0, as a result of which no underwriting deficit arises at time zero.

With Allowance for Taxes

Consider the effect of taxes on the internal rate of return method. Profit of $\tilde{Q}_{t+1}$ in period t is reduced by $\tau\tilde{Q}_{t+1}$. Recall equation (32) as an expression of gross profit, and rearrange it thus:

$$\tilde{Q}_{t+1} = i_{At}\,[K_t + R_t^C - (R_t^P - P_t)] - i_{Lt}R_t^C + i_{Ft}\,(R_t^P - P_t) + v_{t+1}\,(C_{t+1} + E_{t+1}) + K_t\tilde{z}_{t+1} \tag{54}$$

$$= i_{At}\,[A_t + P_t] - i_{Lt}R_t^C + i_{Ft}\,(R_t^P - P_t) + v_{t+1}(C_{t+1} + E_{t+1}) + K_t\tilde{z}_{t+1}, \tag{55}$$

by equations (23) and (26). Now use equation (35) to rewrite equation (54) in terms of technical reserves:

$$\tilde{Q}_{t+1} = i_{At}\,[(1+\theta_t)\,R_t^C - (R_t^P - P_t)] - i_{Lt}\,R_t^C + i_{Ft}\,(R_t^P - P_t) + v_{t+1}\,(C_{t+1} + E_{t+1}) + K_t\,\tilde{z}_{t+1} \tag{56}$$

$$= K_t(i_{It} + \tilde{z}_{t+1}) + v_{t+1}(C_{t+1} + E_{t+1}), \tag{57}$$

by equations (35) and (49). Then, when equation (45) holds, profit net of tax is

$$(1-\tau)\ \tilde{Q}_{t+1} = K_t\ [i_{It}+(1-\tau)\ \tilde{z}_{t+1}] + (1-\tau)\nu_{t+1}(C_{t+1}+E_{t+1}) - \tau\ i_{It}K_t$$

$$= K_t[i_{It}+(1-\tau)\tilde{z}_{t+1}] + R_t^C[(1-\tau)\ \nu_{t+1}(C_{t+1}+E_{t+1})/R_t^C - \tau\ i_{It}\theta_t] \text{ (by equation [35])} \quad (58)$$

$$= K_t\ [i_{It} + (1-\tau)\ \tilde{z}_{t+1}], \quad (59)$$

by (47) in the case (39) and (40) (or [41]) hold in addition to (45).

Equation (58) says essentially no more than the reasoning which lead to equation (47). Of the two members on the right side of equation (58), the first has an expected value equal to the emerging profit in the absence of taxes and reserve loading factors; the second consists of tax on this profit, neutralized (in expected value) by additional profit (net of tax) generated by the release of reserve loading factors. Thus, expected net profit in each period amounts to a return of i_I (by the previous section, $i_{It} = i_I$ under the above assumptions) on outstanding capital. Consequently, the net present value of all expected cash flows to shareholders, including both income and capital, must be zero as required by the internal rate of return method.

Since the discount rate i_I is the capital asset pricing model rate (previous section), the same results will be obtained if each of the flows involved is decomposed into its components and each component discounted at its own specific CAPM rate. This is precisely what is done by the Myers-Cohn method in equation (17). Note that the final member of (17), and its interpretation given toward the end of its corresponding section, is justified by Myers Theorem.

Thus it is shown that the Myers-Cohn and internal rate of return premiums are equal in the presence of taxes, provided still that equations (39), (40) (or [41]), (45), and (46) hold. The results of this and the previous sections may be summarized in the following theorem.

Theorem 1

Suppose that all the following conditions hold:

(a) An insurer computes technical reserves in accordance with the specifications of the section on cash flows (see equations [24] and [47]);

(b) Premiums are computed so that no profit emerges immediately on underwriting, that is, $R_0 = 0$;

(c) Rates of tax and rates of return on assets and liabilities are constant from period to period as in equations (39) and (45) ;

(d) The insurer's capital structure remains constant from period to period in the sense of equation (40).

Then the Myers-Cohn and internal rate of return methods, described by equations (17) and (19), respectively, yield identical premiums. Under these conditions, the returns on equity emerging period by period are all the same and equal to that required by the capital asset pricing model taking into account the insurer's capital structure.■

It is interesting to consider the condition (47), and particularly the fact that it is not the conventional assumption in this field. For example, note that Myers and Cohn assume (1971) that "underwriting profits are accrued as losses are paid. Thus, if 20 percent of losses are paid at the beginning of a period, 20 percent of underwriting profits are assumed to flow to the Company" (p. 68). This assumption is quite different since it is based on underwriting profits which disregard investment income in loss reserving. However, even if investment income is not disregarded, the assumption which runs parallel to that of Myers and Cohn still differs substantially from equation (47), since the latter requires release of profits in proportion with loss reserves rather than losses paid to date.

Consider this parallel assumption:

$$\nu_t = \nu, \text{ const., independent of t.} \tag{60}$$

Then expected profit emerging in period t, other than investment income earned by capital (but including investment income earned by reserves), is

$$\begin{aligned} & r_{Lt}\, R_t^C - R_{t+1}^C - (C_{t+1}+E_{t+1}) \\ & = \nu_{t+1}\,(C_{t+1}+E_{t+1}) \text{ (by equation [28])} \\ & = \nu(C_{t+1}+E_{t+1}), \end{aligned} \tag{61}$$

that is, profits emerge in proportion to the payment of claims.

Note that equations (47) and (60) will be satisfied simultaneously if and only if

$$(C_{t+1}+E_{t+1})/R_t^C = [\theta\tau/\nu(1-\tau)] \text{ ROE}, \tag{62}$$

which is independent of t. By equation (24),

$$\begin{aligned} R_t^C &= \sum_{s=t}^{T} r_{Lt:s}^{-1}(1+\nu)(C_{s+1}+E_{s+1}) \\ &= (1+\nu)\sum_{s=t}^{T} (r_L)^{t-s-1}(C_{s+1}+E_{s+1}), \end{aligned} \tag{63}$$

in the case of r_L constant over time.

Now consider the case of a geometric payment pattern:

$$C_s + E_s = c\rho^s \text{ , } s = 1, 2, \ldots, \tag{64}$$

noting that $T = \infty$ now. Substitution of equation (64) in (63) yields

$$R_t^C = c\rho^{t+1}\,(1+\nu)\, r_L^{-1} / [1 - \rho r_L^{-1}]. \tag{65}$$

Thus, the ratio $(C_{t+1} + E_{t+1})/R_t^C$ is independent of t as required by equation (62). Indeed, an appropriate choice of ν will ensure that equation (62) is satisfied. The above reasoning may be summarized in the following theorem.

Theorem 2

Suppose loss reserving is carried out on the basis of equation (24) with (60). If expected claim (and expense) payments form a geometric payment pattern in the sense of equation (64), then (60) with ν suitably chosen is equivalent to equation (47), and Theorem 1 continues to hold if its conditions (b) to (d) hold. ■

Conversely, if the payment pattern is not geometric and loss reserves are based on the "more usual" equation (60), or some variant of it, the result of Theorem 1 will not hold in general; that is, Myers-Cohn and internal rate of return premiums will not be equal in general.

The key to Theorem 2 is the constancy (over t) of the ratio $(C_{t+1} + E_{t+1})/R_t^C$ (see equation [62]) in the geometric case. Since this leads to equality of Myers-Cohn and internal rate of return premiums, it raises a conjecture that the ordering of these two premiums might be dependent on the behavior of the payment-to-reserve ratio, in particular whether it increases or decreases with t.

Fairley and Hill-Modigliani Methods

General

Somewhat more restrictive assumptions are called for in the analysis of the Fairley and the Hill-Modigliani methods. Both of these methods rely on parameters of an ongoing total insurance portfolio (such as k and S) rather than of a specific cohort. These must be steady state parameters in some sense.

Since cohorts are not considered, there is no meaning to be attached to variation of parameters by period. Hence, all tax rates and rates of return will be assumed below to be constant over time as in equations (39) and (45), as will the loss reserve loading factors ν_t.

In addition, premiums are assumed to be paid in full at the commencement of the underwriting period:

$$P_0 = P, \; P_t = 0 \text{ for } t = 1, ..., T. \tag{66}$$

Fairley Method

Begin with the Myers-Cohn formula (17). With the restriction (66) on premiums, it reduces to

$$P = V(C+E, r_L) + V(T^{(u)}, r_F, r_L) + V(T^{(i)}, r_F). \tag{67}$$

First, consider the case of zero tax. Here equation (67) simplifies further:

$$P = V(C+E, r_L), \tag{68}$$

and, by equation (8),

$$\eta = 1 - (C+E)/V(C+E, r_L). \tag{69}$$

Define

$$k = i_L^{-1}[(C+E)/V(C+E, r_L)-1]. \tag{70}$$

Then, by (69) and (70),

$$\eta = -ki_L, \tag{71}$$

which immediately agrees with the Fairley formula (11) in the case $\tau = 0$.

To this point, equation (71) is no more than a notational rearrangement of the basic premium identity (68). Its real content lies in the interpretation of k.

It is possible to relate k to the technical reserves held by the insurer in the stationary state. Consider the position at the beginning of period zero but just after acceptance of premium P. If the insurer's portfolio is in a stationary state at that time, the expected cost of claims in relation to all premiums underwritten in the past, discounted at rate i_L, will be

$$M = \sum_{t=1}^{T} r_L^{-t} \sum_{s=t}^{T} (C_s+E_s)$$

$$= \sum_{s=1}^{T} (C_s+E_s) \sum_{t=1}^{s} r_L^{-t}$$

$$= \sum_{s=1}^{T} (C_s+E_s)\ (1-r_L^{-s})r_L^{-1}/(1-r_L^{-1})$$

$$= i_L^{-1} \left[\sum_{s=1}^{T} (C_s+E_s) - \sum_{s=1}^{T} r_L^{-s}(C_s+E_s) \right].$$

$$= k\ V(C+E,\ r_L) \text{ (by equation [70])} \quad (72)$$

$$= kP, \text{ (by equation [68])} \quad (73)$$

Thus,

$$k = M/P, \quad (74)$$

and it may now be noted that k is the stationary state ratio of technical reserves to written premiums (net of acquisition costs). Strictly, it is the ratio of present value of expected outstanding losses underwritten to annual premiums, taken at the point immediately after accepting an annual premium. In the more practical case in which new premiums are accepted virtually continuously, the last condition would become unnecessary. The ratio would be simply the stationary ratio of expected present value of claims underwritten to annual premiums.

In the case of no taxes, the Myers-Cohn premium requires $v_t = 0$ for all t, in which case equation (63) indicates that the loss reserve is just the present value of expected claims remaining to be paid.

Note, however, that the numerator of the ratio referred to above, based on all claims underwritten, will include unearned premiums as well as loss reserve. The ratio is thus a stationary ratio of technical reserves to annual premiums.

Fairley (1979) defined his funds-generating coefficient k as a loss, rather than technical, reserve to premium ratio, and not specifically as the value of that ratio which would be observed when the insurance portfolio concerned is in a stationary state. In addition, premiums were not expressed as net of acquisition costs.

Now assume a nonzero tax rate:

$$\tau_t = \tau > 0. \quad (75)$$

Then equation (68) must be replaced by (67). Consider the terms $V(T^{(u)}, r_F, r_L)$ and $V(T^{(i)}, r_F)$ in equation (67). Together, $T_t^{(u)}$ and $T_t^{(i)}$ amount to tax on total expected insurance profit Q_t. Recall equation (32) to obtain

$$Q_{t+1} = \underbrace{P_t - (C_{t+1}+E_{t+1}) + (R_t-R_{t+1})}_{\text{Underwriting Profit}} + \underbrace{i_{At}\,(R_t+K_t+P_t)}_{\text{Investment Income}}. \tag{76}$$

By equations (26), (28), and (29),

$$\begin{aligned} T^{(u)}_{t+1} &= \tau[(R_t^C-R_{t+1}^C) - (C_{t+1}+E_{t+1}) - (R_t^P-R_{t+1}^P) + P_t] \\ &= \tau[\nu(C_{t+1}+E_{t+1}) - i_{Lt}\,R_t^C + i_{Ft}\,(R_t^P-P_t)],\ t = 0, 1, 2, \text{etc.} \end{aligned} \tag{77}$$

Under assumption (66), this reduces to

$$\begin{aligned} T^{(u)}_{t+1} &= \tau[\nu\ (C_{t+1}+E_{t+1}) - i_{Lt}\,R_t^C],\ t = 1, 2, \text{etc.}; \\ &= \tau[\nu\ (C_{t+1}+E_{t+1}) - i_{Lt}\,R_t^C - i_{Ft}\,P],\ t = 0. \end{aligned} \tag{78}$$

The investment income component of equation (76) yields

$$\begin{aligned} T^{(i)}_{t+1} &= \tau i_A\ (R_t+K_t+P_t),\ t = 0, 1, 2, \text{etc.} \\ &= \tau i_A\ (1+\theta)\ R_t^C,\ t = 1, 2, \text{etc.} \\ &= \tau i_A\ (1+\theta)\ R_t^C + \tau i_A\ P,\ t = 0. \end{aligned} \tag{79}$$

By equations (78) and (79),

$$\begin{aligned} &V(T^{(u)}, r_F, r_L) + V(T^{(i)}, r_F) \\ &= \tau[\nu V(C+E, r_L) - i_L \sum_{t=0}^{T} r_L^{-t-1} R_t^C \\ &\quad + i_F(1+\theta) \sum_{t=0}^{T} r_F^{-t-1} R'^{C}_t], \end{aligned} \tag{80}$$

where R'^C_t is defined to be the same as R_t^C except based on a rate of return i_F instead of i_L.

Now define

$$S = P/\theta \sum_{t=0}^{T} r_L^{-t-1}\ R_t^{\,C}, \tag{81}$$

$$S' = P/\theta \sum_{t=0}^{T} r_F^{-t-1}\ R_t^{\prime C}. \tag{82}$$

Substitution of equations (80) to (82) into equation (67) yields

$$P = (1 + \tau\nu)\ V(C+E, r_L) + \tau P\ [i_F\ (1 + \theta)/\theta S' - i_L/\theta S]. \tag{83}$$

Note that, if it is required that $R_0 = 0$, then

$$(1+\nu)\ V(C+E, r_L) - P = 0. \tag{84}$$

Substituting equation (84) in (83), one obtains

$$1 + \nu = 1 + \tau\nu + \tau(1+\nu)\ [...];$$

that is,

$$(1+\nu)^{-1} = 1 - [...]\ \tau/(1-\tau). \qquad (85)$$

Combining equations (8) and (84) yields

$$\begin{aligned} \eta &= 1 - (1+\nu)^{-1}\ (C+E)/V(C+E,\ r_L). \\ &= -ki_L + (1+ki_L)\ [\tau/(1-\tau)]\ [...], \end{aligned} \qquad (86)$$

by equations (70) and (85).

Now consider the final pair of brackets. It may be put in the form

$$[...] = i_F/S' + \Delta, \qquad (87)$$

where $\Delta = \theta^{-1}[i_F/S' - i_L/S]$. (88)

It may be noted that $\Delta = 0$ if $i_L = i_F$. By equations (86) and (88),

$$\eta = -ki_L + (1+ki_L)\ \tau i_F/S'(1-\tau) + (1+ki_L)\ \Delta\tau/(1-\tau). \qquad (89)$$

In the case $r_L = r_F$, equation (88) yields $\Delta = 0$, in which case equation (89) reduces to

$$\eta = -ki_L + (1+ki_L)\ \tau i_F/S'(1-\tau). \qquad (90)$$

The parameter k, as defined by equation (70), is not obviously a loss reserve to premium ratio as it is in equation (11). Note, however, that it is a technical reserve to premium ratio in the case $\tau = 0$ (see equation [74]) and consider the effect of changing τ to a positive value. First, the technical reserve increases by a factor of $1 + \nu$ (see previous section). Second, premium P is increased by the same factor (see equation [84]). Thus, k in fact remains the technical reserve to premium ratio. But, as before, it is the stationary state ratio and relates to premium net of acquisition costs.

Some practitioners take the value of k as just the observed, rather than stationary, ratio of loss reserve to premium. If past growth in exposure has been consistently positive, then the observed ratio will be less than its stationary value; if past growth in exposure has been consistently negative, then the observed ratio will be greater than its stationary value.

The effect of this can be seen by rewriting equation (89) in the form

$$\eta = (i_F/S'+\Delta)\ \tau/(1-\tau) + ki_L\ [(i_F/S'+\Delta)\ \tau/(1-\tau) - 1], \qquad (91)$$

which increases with k as

$S' < i_F\ \tau/[1 - \tau(1+\Delta)]$ and decreases with k as $S' > i_F\ \tau/[1 - \tau(1+\Delta)]$. (92)

In most practical cases the > sign holds in equation (92) and so consistent positive growth in past exposure causes the fair premium to be overestimated when k is taken as simply the observed ratio of technical reserve to premium.

Of course, implicit in the above discussion is that the premium entering into the ratio k is the fair premium (net of acquisition costs); and similarly that the reserve is unbiased and properly discounted at rate of return i_L. To the extent that actual premium underwritten or actual reserves have differed from this, taking k as the observed ratio of technical reserve to premium would introduce further distortion in equation (89).

The result (90) resembles the Fairley formula (11), but even in this special case there are differences in both the formula itself and the interpretation of some of its parameters. In the general case, the differences between equations (11) and (89) are that the tax term of (11) is corrected by a factor of $1 + ki_L$; a further correction term involving Δ is added; and Fairley's premium to surplus ratio S is replaced by S', defined by (82). As regards the last difference, one may note that S' is indeed a premium to surplus ratio, but subject to the conditions that the portfolio is growing at rate r_F per annum and loss reserves are discounted at the risk-free rate. This rate of growth conflicts with the requirement of portfolio stationarity in the interpretation of k.

Hill-Modigliani Method

As noted in the section on fair premium rating methods, Hill and Modigliani were concerned with modifications to Fairley's method which recognized different tax rates in respect of different types of asset and different betas as between traded and nontraded stocks. The first of these is elementary and was dealt with in the discussion leading to equation (14). The second is dealt with below.

As in the earlier discussion, suppose that assets A comprise traded stocks T and nontraded stocks N—A = T + N—and that the traded and nontraded stocks are subject to betas of β_T and β_N, respectively. Then the capital asset pricing model yields

$$\begin{aligned}\beta_A &= (T\beta_T + N\beta_N)/(T + N)\\ &= [(T/P)\beta_T + (N/P)\beta_N]/(T/P + N/P). \qquad (93)\end{aligned}$$

As discussed above, k is a loss reserve to premium ratio; hence, it follows that

$$k = (T+N-K)/P = T/P + k_N - 1/S,$$

in the stationary state, where

$$k_N = N/P. \qquad (94)$$

Thus,

$$T/P = k + 1/S - k_N. \qquad (95)$$

Substitution of equations (94) and (95) in (93) yields

$$\beta_A = [(k + 1/S - k_N)\beta_T + k_N\beta_N]/(k + 1/S). \qquad (96)$$

Then, substitution of equation (96) in (13) yields

$$\beta_L = \{[(kS+1) - k_N S]\beta_T + k_N S\beta_N - \beta_E/(1-\tau)\}/kS. \qquad (97)$$

Formula (15) of Hill and Modigliani is the same as this last result except that k in the denominator of (97) has been replaced by $k+k_N$. Thus, the results here are reconciled with Hill and Modigliani if k is defined in such a way that $k+k_N$ = reserve to premium ratio. Note, however, that this discussion involves the same conflict between S and k (and k_N) and other corrections as discussed above.

Summary

It has been demonstrated that the Myers-Cohn and internal rate of return results were identical under certain specific assumptions. The previous section showed that, under further assumptions, the Fairley formula almost emerged, but with some correction and conflict between the assumptions. It was also shown that a formula similar (but not identical) to Hill and Modigliani's could be derived from Myers-Cohn or internal rate of return as a generalization of Fairley, but subject to similar correction and conflict between assumptions. Schematically, these results are represented in Figure 2.

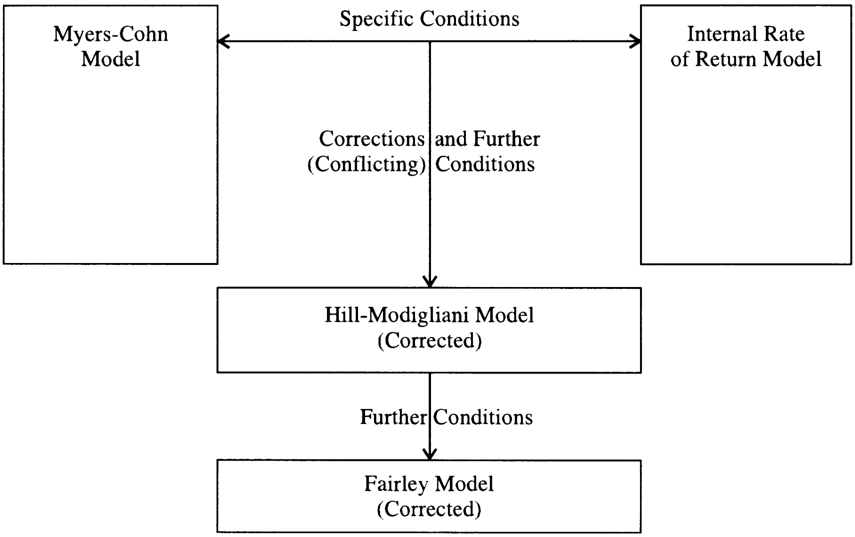

Figure 2
Relations Between Various Models

Appendix

Myers Theorem

Consider an asset of unit value at time zero. Suppose this asset generates a (stochastic) return of $\tilde{r}_s$ in period s = 0, 1, ... t, and that tax on this return becomes payable at rate τ immediately at the end of period s. It is assumed that after-tax returns are reinvested in the same asset. The present value of the expected tax payable at the end of period t can be calculated as if returns are risk free throughout. That is, the required present value is $\tau\, i_{Ft}\, r_{F0:t-1} / r_{F0:t}$.

Proof

The value of the asset at time t+1 is $\tilde{r}_{0t}$, and the appropriate discount factor correspondingly r_{0t}^{-1}. Now income generated in period t will be $\tilde{i}_t\, \tilde{r}_{0:t-1} = (\tilde{r}_t - 1)\, \tilde{r}_{0:t-1}$, and, hence, tax generated is

$$(\tau\, \tilde{r}_t - \tau)\, \tilde{r}_{0:t-1}. \tag{A.1}$$

Taking a standpoint first at the beginning of period t, note that the first term in (A.1) is stochastic while the second is not. Therefore, the value of tax, discounted to time t and conditional on $\tilde{r}_{0:t-1}$, is $(\tau - \tau\, r_{Ft}^{-1})\, \tilde{r}_{0:t-1}$, and discounted to time zero it is $\tau - \tau\, r_{Ft}^{-1}$

$$\begin{aligned} &= (\tau - \tau\, r_{Ft}^{-1})\, r_{F0:t-1}/r_{F0:t-1} \\ &= \tau\, i_{Ft}\, r_{Ft}^{-1}\, r_{F0:t-1}/r_{F0:t-1} \\ &= \tau\, i_{Ft}\, r_{F0:t-1} / r_{F0:t}. \blacksquare \end{aligned}$$

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