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## **Financial Pricing of Insurance in the Multiple-Line Insurance Company**

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### **ABSTRACT**

This paper uses a contingent claims framework to develop a financial pricing model of insurance that overcomes one of the main shortcomings of previous models -- the inability to price insurance by line in a multiple line insurer subject to default risk. The model predicts prices will vary across firms depending upon firm default risk, but within a given insurer prices should not vary after controlling for line-specific liability growth rates. We also analyze an important qualification to this result for insurance groups, where several insurer subsidiaries are owned by a primary insurer or holding company. Empirical tests using data on publicly traded property-liability insurers support the hypotheses: prices vary across firms depending upon overall-firm default risk and the concentration of business among subsidiaries; but within a given firm, prices do not vary by line after adjusting for line-specific liability growth rates.

### **INTRODUCTION**

Since insurance contracts are financial instruments, it seems natural to apply financial models to insurance pricing. Financial pricing models have been developed based on the capital asset pricing model (Biger and Kahane 1978; Fairley 1979), arbitrage pricing theory (Kraus and Ross 1982), capital budgeting principles (Myers and Cohn 1987) and option pricing theory (Merton 1977; Smith 1979; Doherty and Garven 1986; Cummins 1988; and Shimko 1992). Financial models represent a significant advancement over traditional actuarial models because they recognize that insurance prices should be consistent with an asset pricing model or, minimally, avoid the creation of arbitrage opportunities.

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A limitation of the existing financial pricing models is the implicit or explicit assumption that insurers produce only one type of insurance, even though most insurers produce multiple types of coverage (e.g., automobile insurance, general liability insurance, workers' compensation insurance, etc.). The purpose of this paper is to remedy this deficiency in the existing literature by providing a theoretical and empirical analysis of insurance pricing in a multiple line firm. An option pricing approach is adopted to model the insurer's default risk. The standard Black-Scholes model is generalized to incorporate more than one class of liabilities, and pricing formulae are generated for each liability class. The theoretical predictions of the model are tested using data on an extensive sample of publicly traded U.S. property-liability insurers.

Option models of insurance pricing have two primary advantages: First, they explicitly incorporate default risk. This is important given the increase in insurer insolvency rates since the early-1980s (see BarNiv 1990). Second, because of data limitations, the key parameters can be estimated more accurately for option pricing models than for competing models such as the Myers-Cohn (1987) or Kraus-Ross (1982) models.<sup>1</sup>

The standard option pricing model of insurance views the liabilities created by issuing insurance policies as analogous to risky corporate debt. The insurer is assumed to issue an insurance policy in return for a premium payment, analogous to the proceeds of a bond issue. In return, it promises to make a payment to the policyholders at the maturity date of the contract. Using this bond analogy, the value of the insurer's promise to policyholders can be thought of as being like the value of a default risk-free loan in the amount of the promised payment less a put option on the value of the insurer. In reality, however, most insurers issue more than one type of insurance and in this case the analogy with a single debt issue is no longer exact. The problem of pricing multiple classes of debt has been considered by Black and Cox (1976). In their analysis senior debt has priority over junior debt in the event of bankruptcy. However, with multiple lines of insurance, each line has equal priority in the event of bankruptcy (see National Association of Insurance Commissioners 1993), and this is the case investigated in our paper.

In a multiple line insurance company, equity capital is held in a common pool. If one or more lines incur deficits of losses over premiums, the lines in difficulty can draw upon the full amount of the firm's equity capital, including earnings from the "solvent" lines. Given this sharing of resources, it is not obvious how to allocate the cost of equity capital to each line.

There have only been a few prior papers on insurance pricing in a multiple line firm, mostly in the actuarial literature. Nearly all have approached the problem by assuming that

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<sup>1</sup> As explained in detail below, option pricing of insurance requires the estimation of the insurer's overall market volatility parameter, based on monthly or daily stock price data, whereas the Myers-Cohn and Kraus-Ross models require the estimation of one or more beta coefficients measuring the systematic risk of insurance underwriting returns. Due to data limitations, estimation of insurance underwriting betas has relied on quarterly or annual book value data. Betas based on accounting data are likely to be poor proxies for market-value betas, and Cummins and Harrington (1985) report that accounting beta estimates for insurers are highly unstable. Cox and Griepentrog (1988) adapted the pure-play approach of Fuller and Kerr (1981) to estimate divisional costs of capital for insurers but report that the resulting cost of capital estimates are unreliable.

the insurer's equity capital is allocated among lines of business, usually in proportion to each line's share of the insurer's liabilities (see Knuer 1987; Derrig 1989; and D'Arcy and Garven 1990). Prices for a given line of insurance then incorporate an aggregate profit charge equal to the assumed cost of capital for the line multiplied by its assigned equity. This approach ignores the fact that equity capital is shared across divisions and that the risk of the equity may be different from the risk of the line of insurance.

A more appropriate model of multiple line insurance pricing has been developed by Allen (1993), who shows that it is incorrect to allocate capital by line when computing insurance prices because the capital is present to back all of the company's policies and thus is inherently indivisible.<sup>2</sup> Allen's model offers important insights into the multi-line pricing problem. However, it does not incorporate default risk, i.e., it assumes that losses can be larger than expected but can never exceed the insurer's resources. The theoretical development in the present paper combines the option pricing approach with insights drawn from the Allen model to derive a pricing model for a multiple line firm subject to default risk.

Our model implies that it is not appropriate to allocate capital by line; rather, the price of insurance by line is determined by the overall risk of the firm and the line-specific liability growth rates. Thus, prices are predicted to vary across firms depending upon firm default risk, but prices for different lines of business within a given firm are not expected to vary after controlling for liability growth rates by line. An important qualification to this general result is provided by the existence of insurance groups where several separate corporations are owned by a primary insurer or holding company.<sup>3</sup> Under United States corporation law, the owners of the group hold a valuable option; namely, the option to allow a financially troubled subsidiary to fail. The claimants against the insolvent subsidiary cannot reach the assets of other insurers in the group unless they succeed in "piercing the corporate veil," which usually requires showing that the owners engaged in fraud or some other abnormal activity (Easterbrook and Fischel 1985). Although the owners may decide to rescue a failing subsidiary to protect reputation or franchise value, they are under no legal obligation to do so. Thus, we predict that insurance groups in which liabilities are widely dispersed among subsidiaries will command lower prices than unaffiliated single insurers or insurance groups where business is heavily concentrated among the principal affiliates. The empirical results support the hypotheses: Prices vary across firms depending upon overall-firm default risk and the concentration of business among subsidiaries, but within a given firm, prices do not vary by line after adjusting for line-specific liability growth rates.

The remainder of the paper is organized as follows: The next section develops the theoretical model and specifies testable hypotheses. Descriptions of our data sample and definitions of the market-based risk measures and other variables needed to test the

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<sup>2</sup>Allen assumes that insurance is imperfectly diversifiable so that the insurer must hold capital to pay losses that are larger than expected. This assumption is consistent with empirical studies of insurance markets, which show significant degrees of covariability, particularly in high risk lines such as commercial liability insurance and both commercial and residential property insurance (e.g., Harrington 1988).

<sup>3</sup>Groups account for approximately ninety percent of revenues in the property-liability insurance industry (A.M. Best Company 1993).

hypotheses follow. The equation specification, estimation methodology, and test results are presented in the fourth section. The final section concludes the paper.

### THEORETICAL MODEL AND HYPOTHESES

This section develops a theoretical model of insurance pricing in a multiple line firm and then specifies testable hypotheses based on the model.

#### *A Model of Insurance Pricing*

We assume that financial markets are competitive, perfect, and complete and further assume that there are two groups of potential insurance buyers.<sup>4</sup> An insurance company, owned by equity holders, is willing to insure the losses of these two groups of individuals for an appropriate premium  $P_i$ ,  $i=1,2$ .

At time 0, equity (surplus) in amount  $G$  is contributed by the equity holders of the insurer and the insurer receives premiums of  $P_1+P_2$ . The losses of the two groups, denoted  $L_i$ ,  $i = 1, 2$ , are assumed to be payable one period from the present (at time  $t=1$ ). The equity and premium cash flows are invested in marketable securities. As will be discussed below, it is helpful to treat the premiums and the surplus as if they were invested and held in separate asset accounts. The premium and equity accounts evolve over time as (correlated) geometric Brownian motion processes:

$$\begin{aligned} dP_i &= \mu_{P_i} P_i dt + \sigma_{P_i} P_i dz_{P_i} \\ dG &= \mu_G G dt + \sigma_G G dz_G \end{aligned} \quad (1)$$

where  $\mu_{P_i}$ ,  $\mu_G$ ,  $\sigma_{P_i}$ , and  $\sigma_G$  are the drift and diffusion parameters for the premium accounts and the surplus account, respectively. The model allows for correlations across the premium and surplus accounts as follows:  $dz_{P_1} dz_{P_2} = \rho_{P_1 P_2} dt$ ,  $dz_{P_i} dz_G = \rho_{P_i G} dt$  for  $i=1,2$ .<sup>5</sup>

As is often the case in the theoretical options literature, we define an alternative time index,  $\tau = 1-t$ ,  $0 \leq t \leq 1$ , to be the amount of time remaining until the liability payment date (the end of the time period). The market value of the premium and surplus accounts at any time  $\tau$  is equal to:  $P_1(\tau)+P_2(\tau)$ , and  $G(\tau)$ , respectively.

<sup>4</sup>The buyers may be either individuals or business firms. Individuals purchase insurance because they are risk averse. The motivations for the corporate purchase of insurance are discussed in Mayers and Smith (1982) and Shapiro and Titman (1985). The extension to the case of more than two policyholder groups is straightforward.

<sup>5</sup>We could have derived the fundamental results of the model without specifying  $P_1$ ,  $P_2$ , and  $G$  as separate processes. That is, we could just have specified a single asset process  $A = P_1+P_2+G$  and derived our main pricing result equation (14). We choose to specify separate asset processes for  $P_1$ ,  $P_2$ , and  $G$  for two reasons: (1) Doing so enables us to provide an intuitive analysis of the role played by the separate lines of business in a multiple line insurer, i.e., the intermediate steps leading up to equation (14); and (2) using separate asset processes is more general than having a single process for assets because this approach allows the insurer to invest assets generated from equity differently from assets obtained from premiums.

Define  $L_i(\tau)$  to be the market value of the firm's loss liabilities to policyholder class  $i$  at  $\tau$ . The market values of the liabilities are also assumed to evolve according to geometric Brownian motion:

$$dL_i = \mu_{L_i} L_i dt + \sigma_{L_i} L_i dz_{L_i} \quad (2)$$

where  $\mu_{L_i}$  and  $\sigma_{L_i}$  are the drift and diffusion parameters for the liability processes. The liability processes are mutually correlated and are also correlated with the asset processes as follows:  $dz_{L_1} dz_{L_2} = \rho_{L_1 L_2} dt$ ,  $dz_{L_i} dz_G = \rho_{L_i G} dt$ ,  $dz_{P_i} dz_{L_j} = \rho_{P_i L_j} dt$  for  $i=1,2$  and  $j=1,2$ .

The premium, surplus and liability accounts are assumed to be priced according to the intertemporal capital asset pricing model, the ICAPM.<sup>6</sup> The ICAPM implies the following relationships regarding the expected rates of return, i.e., the drift parameters for the various Brownian motion processes:

$$\begin{aligned} \mu_{P_i} &= r_f + \pi_{P_i}, \text{ for premium accounts } i=1,2 \\ \mu_G &= r_f + \pi_G, \text{ and} \\ \mu_{L_i} &= r_{L_i} + \pi_{L_i} \text{ for liability classes } i=1,2 \end{aligned} \quad (3)$$

where  $r_f$  = risk free rate of return,

$r_{L_i}$  = inflation rate for liability class  $i$ ,  $i=1,2$ ,<sup>7</sup> and

$\pi_j$  = the market risk premium for process  $j=P_i$ ,  $L_i$ , and  $G$ ,  $i=1,2$ .

The market risk premium,  $\pi_j$ , is the risk charge investors demand for bearing undiversifiable risk. According to the ICAPM assumption,  $\pi_j$  equals

$$\pi_j = \rho_{jm} \left( \frac{\sigma_j}{\sigma_m} \right) (\mu_m - r_f) \quad (4)$$

where  $\mu_m$ ,  $\sigma_m$  are the drift and diffusion parameters of the Brownian motion process for the market portfolio, respectively, and  $\rho_{jm}$  is the instantaneous correlation coefficient between the

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<sup>6</sup>Using the ICAPM is a standard assumption often made in the finance literature to determine the market price of risk for the underlying optioned asset. In fact, the original Black-Scholes model (1973) uses the single period Sharpe-Lintner-Mossin CAPM model to determine the price of options on an underlying stock. It has been previously been applied in the financial pricing of insurance literature by Cummins (1988) and Cummins and Danzon (1997).

<sup>7</sup>The model is general enough to accommodate cases when the inflation rate for liability class  $i$  is not equal to the economy wide inflation rate. The liability inflation rate will, in general, be a function of both frequency and severity growth rates.

Brownian motion process for the market portfolio and asset class  $j$ , where  $j=P_i$ ,  $L_i$ , and  $G$ , and  $i=1,2$ .

Given this notation, two cases are considered. The first case assumes the equity holders of the insurance company have unlimited liability (no default risk). We consider the unlimited liability case because it provides insights into the insurance pricing process that are helpful in the second case, which extends the analysis to the more realistic situation where equity holders have limited liability and insurance is subject to default risk. The unlimited liability case is also of interest as a model of Lloyd's of London.

*Insurance pricing with unlimited liability.* In the unlimited liability case, if the market value of the insurer's assets (the premium accounts plus the equity account) at time 1 is less than the market value of liabilities, the equity holders agree to make up the deficit from their own resources which are assumed to be adequate to cover any potential shortfall. In this context, the initial capital contribution  $G$  is somewhat analogous to the margin deposits that brokerage houses require from investors when they take positions in futures or forward contracts, i.e., the equity contribution is considered "good faith" money to demonstrate the investor's intention and ability to satisfy the obligations of the contract. The insurance company is somewhat like the brokerage house:<sup>8</sup> it invests the surplus (or margin) in interest or dividend paying assets. Any losses not covered by the premiums and the investment income they earn will be made up by the equity holders from the surplus account held by the insurance company. Any additional funds needed to cover loss payments in excess of the insurer's assets — i.e., in excess of both the premium and surplus (margin) accounts — will be funded by additional contributions from the equity holders.

The assumption of unlimited liability for the equity holders means that we can consider the firm division by division.<sup>9</sup> Therefore, the premium paid by policyholders in a given line of business will equal the market value at the beginning of the time period ( $\tau=1$ ) of the policyholders' claim on the assets of the insurer dedicated to division  $i$ ,

$$PH_i(1) = P_i(1) - EH_i(1) \quad (5)$$

where  $PH_i(1)$  = the market value of the policyholders' claim on the firm;

$P_i(1)$  = the premium account; and

$EH_i(1)$  = the value of the equity holders' claim on division  $i$ .

All quantities in equation (5) are evaluated at time to expiration  $\tau=1$ . Notice that  $EH_i(1)$  can be positive, zero, or negative.

To determine the value of the equity holders' claim on division  $i$ ,  $EH_i(\tau)$ , consider the potential cash flows to or from the equity holders at the end of the time period. In states of the world where the premium account is larger than the losses payable to policyholders, the

<sup>8</sup>In actuality, the brokerage house is an independent agent acting on behalf of the investor. The equity holders in this model are assumed to fully control the insurance company, i.e., there are no managers at the insurance company acting on behalf of the equity holders.

<sup>9</sup>We use the term division synonymously with liability class or line of business.

liability obligations will be paid and the equity holders will receive the residual value. This cash flow can be modeled as the payoff of a call option,  $C_i(P_i, L_i, \tau)$ , with payoff at expiration  $\text{Max}[P_i(0) - L_i(0), 0]$ . We refer to this option as the *divisional call option*. The value of the equity holders' promise to make up any deficit between the premium account and the loss liabilities can be modeled as a put option  $B_i(P_i, L_i, \tau)$ , with payoff at expiration  $\text{MAX}[L_i(0) - P_i(0), 0]$ . We refer to this option as the *divisional put option*. The value of the equityholder's claim on division  $i$  at any time  $\tau$  thus is equal to

$$EH_i(\tau) = C_i(P_i, L_i, \tau) - B_i(P_i, L_i, \tau). \quad (6)$$

Substituting equation (6) into (5) yields

$$PH_i(\tau) = P_i(\tau) - [C_i(P_i, L_i, \tau) - B_i(P_i, L_i, \tau)]. \quad (7)$$

This expression can be further simplified using the put-call parity relationship. Using the terminology of our insurance model, parity requires that the call option the equity holders of the firm own minus the put option they sell to the policyholders is equal to the premium collected minus the discounted expected value of the time 1 loss liability:<sup>10</sup>

$$C_i(P_i, L_i, \tau) - B_i(P_i, L_i, \tau) = P_i(\tau) - L_i e^{-(r_f - r_{L_i})\tau}. \quad (8)$$

Substituting equation (8) into (7) yields:

$$PH_i(\tau) = P_i(\tau) - [P_i(\tau) - L_i e^{-(r_f - r_{L_i})\tau}] = L_i e^{-(r_f - r_{L_i})\tau}. \quad (9)$$

Thus, with unlimited liability, the market value of the policyholders' claim on the firm's assets, and therefore the premium they are willing to pay, is just equal to the present value of the loss liability with the discount rate equal to the risk-free rate minus the liability growth rate.<sup>11</sup> This proposition is not surprising given the analogous results in the risky debt literature (e.g., Merton 1974). Intuitively, the result holds because of the unlimited liability assumption. No matter what state of the world occurs, policyholders always receive the full value of their claim. Since they do not bear any risk, the appropriate discount rate is the risk-

<sup>10</sup>In the standard put-call parity relationship, the exercise price is discounted by the risk-free rate. The discount rate used in this version of the relationship is adjusted by the liability drift parameter  $r_{L_i}$  due to the evolution of the exercise price  $L_i$  over time (see Fischer 1978).

<sup>11</sup>Essentially, the liability start value  $L_i$  accumulates (in an expected value sense) to the policy maturity date ( $t=1$ ,  $\tau=0$ ) at rate  $r_{L_i}$  and is discounted back to the premium payment date ( $t=0$ ,  $\tau=1$ ) at rate  $r_f$ . The discount rate reflects the feature of the model that allows insurance and general inflation to differ. Intuitively, claims incur price inflation at rate  $r_{L_i}$  and are discounted at rate  $r_f$ . If insurance and general inflation are equal, liabilities would be discounted at the real rate of interest  $r_i = r_f - i$ , where  $i$  = the expected rate of general inflation (see Kraus and Ross 1982).

free rate, adjusted in this case for the growth component of the liability drift term to allow for claims inflation.

Equation (7) also implies that the investment strategy undertaken by the firm does not affect the premium paid by the policyholders. In other words, investing the premium account in a risky portfolio will not reduce the price paid by the policyholders. It will, however, affect the risk and return characteristics of the equity holders' payoffs. This result amounts to a type of insurance Modigliani-Miller theorem — assets are valued correctly by financial markets, and value cannot be created or destroyed by the investment policies of insurance firms.

Defining the risk-adjusted discount rate as the discount rate which sets the present value of the liability equal to the policyholders' claim on the firm divided by the expected liability payment (see Merton 1974), we obtain:

$$r_{d_i} = -\frac{1}{\tau} \ln\left(\frac{PH(\tau)_i}{L_i}\right) = r_f - r_{L_i} \quad (10)$$

where  $r_{d_i}$  = the risk-adjusted discount rate for claims of type  $i$ . Thus, the risk-adjusted discount rate for line of business  $i$  when equity holders have unlimited liability is just the risk-free rate minus the growth component of the liability drift term,  $(r_f - r_{L_i})$ .

*Insurance pricing with limited liability.* Now assume that equity holders have limited liability, i.e., equity holders are only liable to pay losses until the assets of the company have been depleted. In the event there are remaining losses to be paid, the equity holders declare bankruptcy and turn the assets of the firm over to the policyholders. In a competitive market with complete information, policyholders will take this limited liability position into account in deciding how much they are willing to pay for the insurance contract.

To determine the value of the equity holders' claim on line of business  $i$  in this case, consider the potential cash flows to or from the equity holders at time 1. In states of the world where the premium account exceeds the losses payable to policyholders in line of business  $i$ , the liability obligations will be paid and the equity holders will receive the residual value. As in the previous section, this contingent cash flow can be modeled as a call option,  $C_i(P_i(\tau), L_i(\tau), \tau)$ . The second possibility is that there will be insufficient funds in the premium account to cover all the liabilities so the equity holders will be required to liquidate part or all of the equity account to make up the difference. This cash flow can be modeled as a put option  $B_i(P_i(\tau), L_i(\tau), \tau)$  sold to the policyholders in line  $i$  by the equity holders.

However, in the limited liability case, if the value of the surplus account does not cover the total shortfall of the firm, the equity holders have the option to declare bankruptcy and default on the remaining loss payments. The value at time  $\tau$  of this potential cash flow can be modeled as a put option known as the insolvency put,  $I(\sum P_i(\tau) + G(\tau), \sum L_i(\tau), \tau) = I(A(\tau), L(\tau), \tau)$ . At expiration ( $\tau = 0$ ), the policyholders receive the value of liabilities less the value of the insolvency put, or  $L(0) - \text{Max}[L(0) - A(0), 0]$ . That is, if liabilities exceed assets at the expiration date of the policies, the equity holders turn over the assets to the policyholders and walk away with no further liability.

We thus conceptualize the multiple line insurer as characterized by two different types of put options. The first type of put,  $B_i(P_i(\tau), L_i(\tau), \tau)$ , which is owned by the policyholders of a given line of business  $i$ , gives the policyholders the right to access the net resources of the firm if liabilities for that line exceed premiums for that line at the expiration date. This put does not have unlimited value but rather is limited by the total resources of the firm. If the total value of liabilities from all lines exceeds the value of assets at the expiration date, the equity holders have the right to default. The value of this right is expressed by the insolvency put  $I(A(\tau), L(\tau), \tau)$ , which is owned by equity holders and reflects their limited liability interest in the firm.

The value of the insolvency put depends upon the firm's total assets and liabilities ( $A$  and  $L$ ), a risk-free drift parameter of  $r_f - r_L$ , and risk parameter  $\sigma_x$  where  $x = A/L$  (see Appendix 1 for the derivation of  $\sigma_x$  and the insolvency put option). The overall liability growth rate  $r_L$  is the weighted average of the line-specific growth rates, i.e.,  $\sum w_{L_i} r_{L_i}$ , where

$$w_{L_i} = \frac{L_i}{\sum_i L_i}. \quad (11)$$

In order to allocate the cost of the insolvency put option to the different lines of business, we need an assumption about the priority in bankruptcy of the various lines of business. We assume that policyholders divide the assets of an insolvent insurer according to an *equal priority rule* which divides the assets of the firm among the policyholders according to the value of the liability claims they hold against the firm. Therefore, each class of policyholders will receive proportion  $w_{L_i}$  of the total assets of the firm in the event of default. The equal priority rule is consistent with insurance bankruptcy laws (see National Association of Insurance Commissioners 1993) and also with the prior academic literature on insurance insolvencies (e.g., Cummins and Danzon 1997). Other types of priority rules could straightforwardly be incorporated in this framework. Using the equal priority rule, the value of the equity holders' claim on line of business  $i$  equals

$$EH_i(\tau) = C_i(P_i, L_i, \tau) - B_i(P_i, L_i, \tau) + w_{L_i} I(A, L, \tau). \quad (12)$$

Now using equations (5) and (12), the value of the policyholders' claim on the firm in line of business  $i$  is

$$PH_i(\tau) = P_i(\tau) - [C_i(P_i, L_i, \tau) - B_i(P_i, L_i, \tau) + w_{L_i} I(A, L, \tau)]. \quad (13)$$

Recalling the parity relationship between the divisional call and put options, equation (13) reduces to

$$PH_i(\tau) = L_i e^{-(r_f - r_{L_i})\tau} - w_{L_i} I(A, L, \tau). \quad (14)$$

Equation (14) says that the value of the policyholders' claim for line of business  $i$  is equal to the risk-free discounted value of the claim minus line  $i$ 's share of the insolvency put option.<sup>12</sup> This is the fundamental insurance pricing relationship derived in this paper. The premium for each policyholder class (line of business) is determined by evaluating equation (14) at the beginning of the time period,  $\tau=1$ , such that the premium paid by each class (the market value of its claim on the firm's assets) equals the present value of expected losses for the class minus its share of the expected loss due to insolvency (the insolvency put).<sup>13</sup>

Equation (14) is consistent with other financial pricing models of insurance as risky debt (e.g., Doherty and Garven 1986; Cummins 1988). However, it generalizes these models, which assume that the insurer has only one line of business, to incorporate multiple lines of business. Equation (14) also overcomes the major shortcoming of other prior multiple line insurance pricing models of insurance that imply that the price of insurance for a given line of business is a function of the amount of equity allocated to that line (Myers and Cohn 1987; and Kneur 1987). The latter authors argue that equity must be allocated to various lines of business in order to determine the fair value of insurance for a particular line. Our analysis shows that such an allocation would be inconsistent with price determination in informationally efficient, competitive insurance markets.

What is important in determining fair insurance premiums is the residual risks that policyholders face. The allocation of surplus to a particular line of business implies that specific lines of business do not have access to the equity capital supporting other lines. This is not the case in practice. The insurer's equity capital provides a cushion against unfavorable realized states of the world and is available to any line of business where it is needed. It is the total amount of equity that the company has and the payouts policyholders can expect from the company that determine the fair market value of insurance.

This prediction must be qualified in the case of insurance groups, which consist of several insurers operating under common ownership. Consider the case of two insurance enterprises with identical asset portfolios, liability portfolios, and business writings. The only difference between the two organizations is that one operates as a single corporate entity with no affiliates or subsidiaries, whereas the second consists of a parent organization with one

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<sup>12</sup>The risk-adjusted discount rate,  $r_{d_i}$ , is the discount rate which sets

$$L_i e^{-r_{d_i} \tau} = PH(\tau).$$

In the limited liability case, taking the logs of both sides and solving for  $r_{d_i}$  it can be seen that the risk-adjusted discount rate is greater than the risk-free interest rate net of the line-specific liability growth rate.

<sup>13</sup>Notice that the premiums implied by equation (14) do not depend on the asset or liability risk premia specified in equations (3) and (4). This is due primarily to the assumption of market completeness, which allows us to eliminate the market risk premia through hedging arguments (see Appendix). Elimination of the risk premia also reflects the underlying assumption that all arbitrage opportunities will be eliminated in a competitive market. Our result is similar to the general Black-Scholes result that the price of an option does not depend on the market risk premium of the underlying stock. It is also consistent with the risky debt literature in finance (Merton 1974) and the prior literature on option pricing models in insurance (Cummins 1988). Equation (14) could easily be modified to incorporate expenses, using standard actuarial formulas. However, we believe it is clearer, both theoretically and empirically, to net out expenses in conducting our analysis. This approach also is consistent with the prior literature (Cummins and Danzon 1997; Sommer 1996).

or more subsidiaries. The latter firm (the insurance group) holds a valuable option that is not available to the freestanding insurer, namely the option to permit a subsidiary that is experiencing financial difficulties to fail. The option's value stems from corporation law, which does not permit the creditors of a group member to reach the assets of a parent or other group member unless they succeed in "piercing the corporate veil," which usually requires the presence of fraud or some other type of misconduct (see Easterbrook and Fischel 1985).<sup>14</sup>

Of course, the parent or other group members might voluntarily come to the aid of a group member facing insolvency to protect the reputation or franchise value of the group. However, as long as the probability of a bailout is less than 1, debt claims of the group are worth less to its debt holders than the debt claims of the otherwise identical freestanding insurer are to that insurer's debt holders. Thus, the price of insurance sold by groups should tend to be less than the price of insurance offered by otherwise identical firms which write all of their business out of a single corporate entity; and other things equal, prices in a cross-section of groups are likely to vary with the degree of dispersion of business across group members.

Another real-world qualification of our principal theoretical result is provided by the existence of insurance guaranty funds, which are designed to protect claimants against insolvent insurers by making up the shortfall between assets and liabilities. (Guaranty funds are state-mandated but industry-operated associations that obtain funds to pay claims by making assessments against the remaining solvent insurers.) If the protection provided by guaranty funds were complete, then we would not expect insurance prices to vary cross-sectionally with the insolvency put value. However, we argue that guaranty fund protection is far from complete. Claimants against insolvent insurers encounter delays in receiving claim payments, tend to incur higher transactions costs than for claims against solvent insurers, and forfeit the benefit of services the insurer would have provided beyond paying the claim. All insurance guaranty funds include an upper limit on the amount payable to any claimant (\$300,000 in most states but as low as \$100,000 in some states), and questions have been raised about the general adequacy of the funds' resources (see U.S., General Accounting Office 1992). Thus, even in the presence of guaranty funds, we predict that insurance will be priced in the market as risky corporate debt.

### *Hypotheses*

This section develops testable hypotheses about the pricing of insurance in multiple line firms based on our theoretical model. The price of insurance is usually measured by the unit price, or ratio of premiums to expected losses to be paid at the end of the time period (e.g., Harrington 1988). Using the notation developed in the previous section, the premium-to-liability ratio is defined as:

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<sup>14</sup>Although insurance groups are common in property-liability insurance, it is unusual for a member of a group to write only one line of insurance. Group members usually tend to be multiple-line companies, although they may specialize in a particular subset of lines, such as personal vs. commercial, or in particular geographical regions.

$$PR_{ij} = \frac{P_{ij}(1)}{E(L_{ij}(0))} = \frac{P_{ij}}{L_{ij}e^{r_{L_i}\tau}} \quad (15)$$

where  $PR_{ij}$  = the unit price of insurance for company  $j$  in line  $i$ ;  
 $P_{ij}(1)$  =  $P_{ij}$  = the premium paid at time  $t = 0$  (time to expiration  $\tau = 1$ ) for company  $j$ , line  $i$ ;  
 $L_{ij}(0)$  = the value of liabilities at the policy expiration date (time to expiration  $\tau = 0$ );  
 $L_{ij}(1)$  =  $L_{ij}$  = the starting value at  $\tau = 1$  for liability process  $i$  in company  $j$ ;  
 $r_{L_{ij}}$  = the instantaneous expected liability growth rate for line  $i$  in company  $j$ ;  
 $\tau$  = time until maturity; and  
 $E$  = the expectation operator.

Using the formula for the competitive-market premium, equation (14), we obtain:<sup>15</sup>

$$\frac{P_i}{L_i e^{r_{L_i}\tau}} = \frac{L_i e^{-(r_f - r_{L_i})\tau} - w_{L_i} I(A, L, \tau)}{L_i e^{r_{L_i}\tau}} \quad (16)$$

where  $r_f$  = the risk-free rate, and  $I(A, L, \tau)$  = the insolvency put for the entire insurance company, as above. In equation (16), the company subscript ( $j$ ) has been suppressed to simplify the notation.

Differentiating equation (16) with respect to the firm's overall risk parameter  $\sigma_x$  yields:

$$\frac{\partial PR_i}{\partial \sigma_x} = -w_{L_i} \frac{\partial I}{\partial \sigma_x} < 0, \quad (17)$$

i.e., the unit price of insurance is inversely related to the insurer's risk parameter,  $\sigma_x$ . Differentiating with respect to the asset-to-liability ratio ( $x$ ) yields the related prediction that the unit price of insurance will be directly related to  $x$ . These results yield the first testable hypothesis.

*Hypothesis 1: In an informationally efficient, competitive insurance market, the price of insurance will be inversely related to firm default risk.*

Hypothesis 1 is consistent with the existing literature on the pricing of insurance in firms subject to default risk (e.g., Cummins 1988). However, our model also yields predictions about insurance prices that differ markedly from conventional predictions. These predictions

<sup>15</sup>In equation (14) and the preceding derivation, the value of the claim on the firm by policyholders of class  $i$  is denoted  $PH_i(\tau)$ . Actually,  $PH_i(\tau)$  and  $P_i(\tau)$  are identical at the date the premium is determined ( $\tau=1$ ), i.e.,  $PH_i(1) = P_i(1)$ . The distinction between  $PH_i(\tau)$  and  $P_i(\tau)$  was helpful earlier in discussing the divisional put value and deriving equation (14). The  $PH$  notation is now dropped, and  $P_i(\tau)$  is used as the price of insurance in the remainder of the paper.

are reflected in our second hypothesis, which concerns the relationship among prices of insurance across lines of business for a given insurer. Recalling that  $w_{L_i} = L_i/L$ , equation (16) can be simplified as follows:

$$\frac{P_i}{L_i e^{r_{L_i} \tau}} = e^{-r_f \tau} - \frac{I(A, L, \tau)}{L} e^{-r_{L_i} \tau}. \quad (18)$$

From equation (18) we can see that differences in the premium-to-liability ratios across lines of business for a given insurer can be explained by differences in the expected line-specific liability growth rates and the size of the insolvency put relative to the total liabilities of the firm. Thus, the model predicts that premium-to-liability ratios will vary across lines as a function of the line-specific growth rates, but will not vary with respect to line-specific risk.

This discussion suggests the following hypothesis:

*Hypothesis 2: The difference between premium-to-liability ratios across lines for a given insurer will be equal to zero after controlling for overall firm risk and the expected liability growth rates of the lines of business.*

Hypothesis 2 is much different from the usual hypothesis in the insurance literature. Many prior insurance pricing models predict that the differences between premium-to-liability ratios across lines of business for a given insurer are a function not only of the line-specific liability growth rate but also of the riskiness of the line of business. This prediction arises in both the actuarial literature (e.g. Daykin, Pentikainen and Pesonen 1994) and the financial literature (e.g., Myers and Cohn 1987; Derrig 1989). In general, these results are obtained whenever there is either some explicit or implicit allocation of equity capital to individual policies or lines of business. The model presented in this paper, on the other hand, implies that prices reflecting the allocation of equity capital by line of business are likely to be inconsistent with prices in informationally efficient, competitive insurance markets. It is the riskiness of the *entire* value of the firm that is relevant in determining the premium to liability ratio for a given line, through its effect on the term  $I(A, L, \tau)$  in equation (18).<sup>16</sup>

As mentioned above, however, this hypothesis must be qualified for insurance groups, where liabilities and equity are dispersed among two or more corporate entities rather than being held in a single firm as assumed in the preceding analysis. Here the allocation of equity among members of the group does matter because of the group's option to permit an individual subsidiary to fail. This suggests a third hypothesis:

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<sup>16</sup>Of course, the riskiness of the entire value of the firm ( $\sigma_x$ ) is a function of the risk parameter of the firm's asset process ( $\sigma_A$ ), the risk parameters of the individual lines of business ( $\sigma_{L_i}$ ), and the instantaneous covariances among lines and between lines and assets (e.g.,  $\sigma_{L_i L_j}$ ) (see Appendix). However, the individual risk parameters and covariances do not appear independently in equation (18), only in the expression for  $\sigma_x$ , which is embedded in  $I(A, L, \tau)$ . The effect of the line-specific risk parameter thus differs from that of the line-specific liability growth rate ( $r_{L_i}$ ), which does appear independently in (18).

*Hypothesis 3: The price of insurance will be inversely related to the dispersion of liabilities among subsidiaries that are separate corporations within an insurance group.*

This hypothesis is consistent with the prior theoretical literature on option pricing. E.g., Merton (1973), shows that a portfolio of warrants is worth more than a warrant on a portfolio consisting of the underlying stocks on which the warrants are written. Analogously, a portfolio of insolvency puts on the members of a group has a higher value than a put on a portfolio consisting of the stocks of the group members. Thus, dispersion of business among corporate entities lowers the value of insurance to policyholders provided that there is a non-negligible chance that the group's owners will successfully exercise their option to allow a subsidiary to default.

To further test the theoretical model, differentiating (18) with respect to  $r_{L_i}$  yields:

$$\frac{\partial PR_i}{\partial r_{L_i}} = -\frac{\partial I(A, L, t)}{\partial r_{L_i}} e^{-r_{L_i} \tau} + \frac{I(A, L, t)}{L} \tau e^{-r_{L_i} \tau}. \quad (19)$$

The sign of expression (19) is ambiguous since the first term is negative (i.e., the value of the insolvency put increases with the expected liability growth rate); while the second term will always be positive.<sup>17</sup> To gain further insight, substituting for the partial derivative of the insolvency put with respect to the  $r_{L_i}$  yields

$$\frac{\partial PR_i}{\partial r_{L_i}} = \tau e^{-r_{L_i} \tau} \left[ -\frac{A}{L} N(-d_1) + (1 - w_i) e^{-(r_f - r_{L_i})} N(-d_2) \right] \quad (20)$$

where  $d_1 = [\ln(A/L) + (r_f - r_{L_i} + 0.5\sigma_x^2)\tau] / (\sigma_x \sqrt{\tau})$ ,  $d_2 = d_1 - \sigma_x \sqrt{\tau}$ , and  $N(\bullet)$  = the standard normal distribution function. The sign of expression (20) is still ambiguous but is more likely to be negative for lines that represent relatively high proportions of total liabilities.<sup>18</sup> For the monoline insurer ( $w_i=1$ ), (20) is unambiguously negative because the increased liability growth rate raises the value of the insolvency put which has the effect of decreasing the premium relative to the expected liability payment. Based upon this discussion, we predict changes in the liability growth rates will be negatively related to the premium ratio for the given line of business.

<sup>17</sup>In differentiating (18), it is important to note that the risk-free drift term in this model is  $r_f - r_{L_i}$ . Thus, the effect of  $r_{L_i}$  on the put is the opposite of the effect of  $r_f$  in the standard Black-Scholes model.

<sup>18</sup>Equation (20) will only be positive in situations when the line of business being considered is small relative to the total business of the firm and is currently experiencing very high growth rates. Intuitively this makes sense as the high growth rate is going to raise the premium paid by the policyholders in the line of business but will have a negligible effect on the value of the insolvency put due to the small effect it will have on increasing the total loss payments of the firm.

## DATA AND VARIABLE DEFINITIONS

This section discusses our data base and specifies the key variables needed to conduct the empirical tests — market-based measures of insurance risk and the price of insurance.

### *The Sample*

Because our hypotheses require market value estimates of assets, equity, and firm risk, our sample consists of publicly traded stock insurance companies. Ninety publicly traded insurers are included, for the time period 1988 to 1992.<sup>19</sup> This is essentially the universe of traded stock insurers that met our selection criteria, i.e., that the firm be either a property/liability insurance company or a multi-line insurer with at least 25 percent of its premium revenues in property/liability insurance. Data on stock returns were obtained from the Center for Research on Securities Pricing (CRSP) tapes for stocks traded on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and the NASDAQ. Financial statement data were obtained from the 10-K reports and the A.M. Best Company data tapes.<sup>20</sup> Specific data items are discussed in more detail in conjunction with the definition of variables used in the analysis.

### *Market-Based Estimates of Firm Risk*

The model presented in theoretical section suggests the price of insurance will be a function of the riskiness of the issuing firm. To test this hypothesis empirically, we extend the Ronn and Verma (1986; 1989) option pricing methodology to derive market measures of the riskiness of the insurer.<sup>21</sup> The risk measures are then used to test the implications of the theoretical model. Our use of market derived measures of firm-specific risk are significant for two reasons. First, Cummins and Danzon (1997) and Sommer (1996) have already investigated the effect various balance sheet risk measures have on the price of insurance. Second, the market derived measure of risk we use in this paper is theoretically consistent with the model presented in section 2. This is appealing since we can then correctly incorporate the nonlinear and interactive effect that the volatility of the asset risk of the firm and leverage of the firm will have on the price of insurance.

In applying the Ronn and Verma methodology, we extend their approach in two important ways. First, our approach allows us to obtain estimates of an insurer's insolvency put which recognize that the insurance company's liabilities evolve as stochastic processes, whereas Ronn and Verma assume that bank liabilities are non-stochastic. Second, we

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<sup>19</sup>Because there were some entries and exits during the sample period, the ninety companies were not available for all years of the sample period. The total number of observations is 315, including 54 companies available for all five years and 45 observations on companies available for only part of the period. Empirical analysis of the complete panel subset yielded qualitatively the same results as the analysis based on the sample of all available observations.

<sup>20</sup>The A.M. Best data tapes report data from the regulatory annual statements filed by insurers with state insurance departments and the National Association of Insurance Commissioners.

<sup>21</sup>Gorton and Santomero (1990) and Flannery and Sorescu (1996) also employ a variant of the option pricing model to derive a market measure for the default risk premia in the rates paid on bank subordinated notes and debentures.

control for potential bias induced by the non-synchronous trading observed in the stock of several of the smaller companies in the sample. Non-synchronous trading can significantly bias equity return volatility (Lo and MacKinlay 1990).

The Ronn and Verma methodology estimates the market value of the assets of the firm,  $A$ , and the implied volatility of the value of the firm,  $\sigma_x$ , by solving the following two simultaneous equations based on the formula for the owners' equity call option:

$$E = AN(d_1) - Le^{-r_x\tau}N(d_2) \quad (21)$$

and

$$\sigma_E = \frac{N(d_1)A}{E} \sigma_x \quad (22)$$

where  $E$  = the market value of equity,

$A$  = the market value of assets,

$L$  = the nominal (undiscounted) value of liabilities,

$x$  = the asset-to-liability ratio =  $A/L$ ,

$\tau$  = time until payment of loss liabilities,

$r_x$  = the risk-free interest rate net of the growth rates of the insurer's liabilities (see equation (23)), i.e., the risk-neutralized drift term on the process  $x = A/L$ ,

$\sigma_x$  = the diffusion parameter of the process  $x = A/L$ , a function of the diffusion and covariance parameters of the premium, surplus, and liability processes (see Appendix 1),

$\sigma_E$  = the standard deviation of the firm's equity returns,

and  $d_1$  and  $d_2$  are defined above, following equation (20). The estimation of the other parameters in equations (21) and (22) is discussed below. Our approach yields four major market-based measures of the riskiness of the firm —  $\sigma_E$ ,  $\sigma_x$ , the asset-to-liability ratio  $x = A/L$ , and the value of the insolvency put,  $I(A, L, \tau)$ .

The equity return standard deviation ( $\sigma_E$ ) was estimated using both daily and weekly data. The daily standard deviations of equity returns are based on the most recent 200 trading days before the end of the year, while the weekly estimates are based on the most recent forty weeks of weekly return data prior to the end of the year.<sup>22</sup> The daily measures were annualized by multiplying the daily standard deviation by the square root of the number of trading days during the year, and the weekly measures were annualized by multiplying by the square root of fifty-two weeks.

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<sup>22</sup>Theoretically, the weekly and daily measures of annualized volatility should be identical. Both methods are used here to check the robustness of the estimates. The time period of forty weeks (200 trading days) was chosen to balance the need to use a long enough series to obtain a reasonable estimate of the variability of the return series without using returns from time periods that no longer reflect the current economic environment or company characteristics. The length of the series used here is similar to that used by other authors (Ronn and Verma 1986; Cordell and King 1995).

Two methods were used to estimate  $\sigma_E$ . The first makes no adjustment for non-synchronous trading, while the second adjusts for non-synchronous trading using the procedure discussed in Smith (1994). Smith's procedure provides estimates of the true standard deviation of equity returns,  $\sigma_E$ , and the bias which is induced by non-synchronous trading,  $\sigma_S$ .

The market value of equity,  $E$ , for the insurance company was set equal to the market capitalization of the firm as reported in the CRSP data base for December 31 of each study year. The total liabilities of the firm,  $L$ , were obtained from the consolidated balance sheets as reported in the firm's 10-K form.<sup>23</sup> The discount rate,  $r_x$ , for each company is (see Appendix 1 for the derivation):

$$r_x = r_f - [w_{L_1} r_{L_1} + \dots + w_{L_n} r_{L_n}] \quad (23)$$

where  $r_f$  is the risk-free rate,  $r_{L_i}$  is the growth rate for line of business  $i$  and  $w_{L_i}$  is the proportion of the insurer's total reserves in line of business  $i$ . The risk-free rate  $r_f$  is the one-year Treasury yield rate from Coleman, Fisher and Ibbotson (1989) for 1988. For the years 1989 to 1992 the risk-free rate used was the yield to maturity of one-year Treasury strips as reported in *The Wall Street Journal*.

The liability growth rate for each line of business  $i$ ,  $r_{L_i}$ , was estimated as the average five-year growth rate of total industry accident year losses and loss adjustment expenses incurred for each line of business reported in the A.M. Best data tapes.<sup>24</sup> For each study year, five-year growth rates for the period ending on December 31 of the study year were used. The weights,  $w_{L_i}$ , used in equation (23) to construct  $r_x$  vary by insurer and are estimated from the data on incurred losses and loss adjustment expenses by line reported in the A.M. Best data tapes. All major lines of business were used. Smaller lines were grouped together following the line groupings in Schedule P of the regulatory annual statement. The time to maturity,  $\tau$ , was set equal to 1, based on the rationale that regulatory audits are performed annually (see Pennacchi 1987; Ronn and Verma 1986; D'Arcy and Garven 1990).

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<sup>23</sup>The value of liabilities implied by the theoretical model is analogous to the strike price in an option where the strike price is stochastic. The strike price in our insurance model is the time 1 value of liabilities. The expected value of this variable at time 0 is the starting value of the promised liability payments accumulated at the liability growth rate  $r_L$  and discounted at the risk-free rate  $r_f$ . Thus, the appropriate value of  $L$  is the company's estimate at time zero of the nominal liabilities at that time, i.e., the company's stated loss reserves (analogous to the face value of a bond issue). For more details see Cummins (1988). Because many of the firms in the data set also provide financial services other than insurance, they typically carry non-insurance liabilities on their balance sheets. As our estimate of  $L$ , we use the total liability figure reported in the firm's 10-K report, which includes non-insurance liabilities as well as loss reserves.

<sup>24</sup>Industry-wide growth rates are used because they provide a better measure of the *expected* growth rates than company-specific growth rates. Company-specific growth rates tend to be affected by idiosyncratic factors and events that are unlikely to recur in the future. Such effects are diversified out of the industry-wide growth patterns. We allow for the company-specific effects in our analysis by using panel data methods (fixed and random effects models). We also conducted the analysis using the industry-wide growth rates in *loss reserves* rather than losses incurred. This analysis led to similar conclusions with regard to Hypotheses 1, 2, and 3.

### Estimating the Price of Insurance

Two definitions of the price of insurance are used in this study: the premium-to-loss ratio and the economic premium-to-loss ratio.<sup>25</sup> Recall that the *premium-to-liability ratio* implied by the theoretical model is equal to the premiums collected divided by the expected value of losses (see equation (15)). However, even though the company is audited one period after policy issue, claim payout periods for some types of insurance (e.g., liability insurance) span several time periods. To control for differences in price resulting from inter-line differences in payout periods, tests also are conducted using the *economic premium-to-loss ratio*, which is the ratio of the premiums to the expected value of losses discounted at the risk-free rate (Winter 1994). Using actual premiums in the numerator and the riskless present value of losses in the denominator allows us to capture inter-firm differences in prices due to insolvency risk.

More precisely, we define:

$$PR_j = \frac{\sum_{i=1}^N [NPW_{ij} - DIV_{ij} - UEX_{ij}]}{\sum_{i=1}^N [NLI_{ij} + LAE_{ij}]} \quad (24)$$

$$EPR_j = \frac{\sum_{i=1}^N [NPW_{ij} - DIV_{ij} - UEX_{ij}]}{\sum_{i=1}^N [(NLI_{ij} + LAE_{ij}) \times PVF_i]} \quad (25)$$

where  $PR_j$  = the premium-to-loss ratio for company  $j$ ,  
 $EPR_j$  = the economic premium-to-loss ratio for company  $j$ ,  
 $NPW_{ij}$  = net premiums written for line  $i$ , company  $j$ ,  
 $DIV_{ij}$  = policyholder dividends paid for line  $i$ , company  $j$ ,  
 $UEX_{ij}$  = underwriting expenses incurred for line  $i$ , company  $j$ ,  
 $NLI_{ij}$  = net losses incurred for line  $i$ , company  $j$ ,  
 $LAE_{ij}$  = net loss adjustment expenses incurred for line  $i$ , company  $j$ ,  
 $PVF_i$  = present value factor for line  $i$ , and  
 $N$  = the number of lines of insurance used to compute the ratio.

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<sup>25</sup>In discussions regarding the empirical tests, we will be referring to the price of insurance as the premium -to-loss or the economic premium-to-loss ratio. In the theoretical discussion, section 2, we referred to the price of insurance as the premium-to-liability ratio. Our theoretical model is a one period model where we assume all loss payments will be made at the end of the time period, thus there is no distinction between losses and liabilities. However, we make the change in terminology when discussing the empirical tests to recognize the distinction between losses incurred and the liabilities insurers establish to fund future loss payments on losses already incurred but are not paid by the end of the time period.

Because underwriting expenses vary significantly across lines of insurance and default risk pertains to the expected loss component of the premium (the so-called pure premium), underwriting expenses are netted when computing the premium and economic premium-to-loss ratios.

Equations (24) and (25) were calculated for two major line groupings — long and short-tail lines — giving long-tail and short-tail premium and economic premium ratios for each company in each year of the sample period. Lines of business that generally pay 90 percent of claims within three years were considered short-tail lines, while lines that take longer to close are considered long-tailed lines (see Appendix 2 for the lines in each category).<sup>26</sup>

The present value factors in equation (25) require estimates of loss payout tail proportions as well as U.S. Treasury yield curves.<sup>27</sup> The payout tail proportions were estimated using the method prescribed by the Internal Revenue Service for computing loss present values for tax purposes (see Cummins 1990). Data to implement the IRS methodology were obtained from industry-wide regulatory statement data provided in *Best's Aggregates and Averages (1986-1993)*.<sup>28</sup>

Observations with premium or economic premium-to-loss ratios less than zero or greater than five were eliminated. Such extreme ratios tend to be indicative of insurers that are exiting lines of business. This left a sample of seventy-one companies with 315 observations over the time period 1988 to 1992. The companies included in the sample are listed in Appendix 2.

## EMPIRICAL RESULTS

The implications of the model presented in section 2 are investigated by conducting several empirical tests. This section first presents summary statistics for the variables used in the

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<sup>26</sup>Grouping lines into long and short-tail categories is a standard procedure in the insurance economics literature because long and short-tail losses and profits often behave differently whereas intra-category differences are much less pronounced. Grouping has the benefit of preserving degrees of freedom.

<sup>27</sup>The formula for the present value factor is:

$$PVF = \sum_{t=1}^T c_t \left( \frac{1}{1+r_t} \right)^t$$

where PVF = the present value factor,  $c_t$  = the proportion of losses paid at time  $t$  ( $0 < c_t \leq 1$ ),  $\sum_t c_t = 1$ ,  $r_t$  = the discount rate for flows at time  $t$ , and  $T$  = the number of periods in the payout tail.

<sup>28</sup>Data on payout tails by line and by company were not available to us. However, payout tails are not expected to differ significantly across insurers for a given line of business. Using the industry data has the virtue of smoothing out random fluctuations in realized payouts that would not necessarily be incorporated in expectations when pricing. The regulatory statement aggregates some of the minor lines of insurance into composite lines rather than reporting them separately. For instance, aircraft, boiler and machinery, and ocean marine insurance are combined and reported as special liability. For the composite lines, the present value factor calculated using the composite data is applied to each component constituting the composite line. As above, the Treasury yield curves used to discount the cash flow proportions were obtained from Coleman, Fisher and Ibbotson (1989) for 1988 and estimated from Treasury Strips reported in *The Wall Street Journal (WSJ)* for 1989 through 1992. The former source was used for 1988 for convenience; the results would be the same if we had estimated the 1988 yield curve based on WSJ data.

empirical analysis. We then present the results of tests of our hypotheses about price differences across insurers and for different lines of insurance within insurers.

### Summary Statistics

Table 1 - Panel A reports summary statistics for the variables used in the study. The average book value of assets for the firms in the sample is \$7.8 billion and the average market capitalization is \$1.4 billion. The average long-tail premium-to-loss ratio (net of underwriting expenses) is 0.93 and the average long-tail economic premium-to-loss ratio is 1.12. For short-tail lines, the average premium-to-loss and economic premium-to-loss ratios are 1.18 and 1.24, respectively.

**Table 1**  
Panel A: Summary Statistics, 1987-1992

	Symbol	Mean	Standard Deviation	Min	Max
Book Value of Assets (000's)	ASSETS	\$7,789,155	\$16,975,026	\$ 16,300	\$91,987,600
Book Value of Liabilities (000's)	TOTLIABS	\$6,677,785	\$15,269,476	\$ 8,124	\$84,603,100
Market Capitalization (000's)	EQUITY	\$1,406,231	\$2,951,311	\$ 747	\$24,541,308
Book Value Asset-to-Liability Ratio	BVAL	1.3839	0.2391	1.0000	2.2837
Short-Tailed Liability Growth Rate	LIABGROS	4.76%	6.46%	-11.41%	54.30%
Long-Tailed Liability Growth Rate	LIABGROL	8.37%	4.55%	-4.87%	23.91%
Net Interest Rate	RX	0.22%	2.50%	-13.06%	8.07%
Group Herfindahl	COHERF	0.5907	0.2642	0.1373	1.0248
Long-tailed Premium Ratio *	PRL	0.9325	0.2188	0.3551	2.3770
Long-Tailed Economic Premium Ratio*	EPRL	1.1163	0.2732	0.4123	2.8621
Short-Tailed Premium Ratio *	PRS	1.1758	0.4342	0.1620	4.7791
Short-Tailed Economic Premium Ratio*	EPRS	1.2393	0.4560	0.1740	4.9975

Number of observations - 315

Underwriting expenses were deducted from the numerator before calculating the Premium and Economic Premium Ratio variables

Table 1 - Panel B reports summary statistics for the market-based measures of firm risk employed in this study. The unadjusted annualized volatility of the equity returns based on

**Table 1**  
**Panel B - Market-Based Risk Measure Summary Statistics: 1987-1992**

	Symbol	Mean	Standard Deviation	Min	Max
Annualized St. Dev of Equity Returns Based on Daily Data	SIGMA	0.3789	0.2393	0.0701	1.9137
Annualized St. Dev of Equity Returns Controlling for Non- Synchronous Trading Bias	SIGMANT	0.2904	0.1693	0.0214	1.4473
Annualized St. Dev of Equity Returns Based on Weekly Data	WSIGMA	0.3184	0.1831	0.0693	1.5643
Annualized St. Dev of Equity Returns Controlling for Non- Synchronous Trading Bias	WSIGMANT	0.2692	0.1561	0.0204	1.2512
Implied Volatility of Firm Estimated from SIGMA	SIGX	0.1154	0.1012	0.0131	0.8018
Implied Volatility of Firm Estimated from SIGMANT	SIGXNT	0.0860	0.0819	0.0044	0.7972
Implied Volatility of Firm Estimated from WSIGMA	WSIGX	0.0950	0.0861	0.0106	0.8197
Implied Volatility of Firm Estimated from WSIGMANT	WSIGXNT	0.0803	0.0804	0.0072	0.8303
Insolvency Put Estimated from SIGMA (000's)	PUT	\$1,074	\$6,030	\$ 0	\$91,749
Insolvency Put Estimated from SIGMANT (000's)	PUTNT	\$797	\$5,826	\$ 0	\$90,166
Insolvency Put per Liabilities Estimated from SIGMA	LPUT	0.45%	1.78%	0.00%	15.99%
Insolvency Put per Liabilities Estimated from SIGMANT	LPUTNT	0.15%	1.01%	0.00%	10.75%
Market Value of Assets Estimated from SIGMA (000's)	MVA	\$7,995,860	\$16,853,739	\$18,609	\$90,236,964
Market Value of Assets Estimated from SIGMANT	MVANT	\$7,996,193	\$16,855,182	\$18,672	\$90,228,563
Market Value Asset-to-Liability Ratio Estimated from SIGMA	MVAL	1.5281	0.5199	0.9516	3.5392
Market Value Asset-to-Liability Ratio Estimated from	MVALNT	1.5274	0.5174	0.9925	3.5406

Number of observations - 315

daily data is 0.379.<sup>29</sup> The adjustment for non-synchronous trading reduces the mean annualized volatility estimate to 0.290. The average implied volatility of the asset-to-liability ratio,  $\sigma_x$  (SIGX in the table), is 0.115. Controlling for non-synchronous trading reduces this estimate to 0.086. Because of the non-synchronous trading problem, the remainder of the discussion is based on the trading-adjusted volatility estimates.

The implied volatility estimate for the insurers in our sample ( $\sigma_x$ ) is higher than the estimates reported in the literature for commercial banks. For example, Ronn and Verma report the average implied annualized volatility for a sample of 43 banks in 1987 as 0.017. Cordell and King (1995) report volatilities for samples of 302 commercial banks and 173 savings and loans in 1990 of 0.022 and 0.013, respectively. The most likely reason for the higher volatility estimates for insurers is the riskier nature of insurance liabilities. Insurers also invest a higher proportion of their assets in equities than do banks.

The average market value of the insolvency put (based on daily data after adjusting for nonsynchronous trading) is \$797,000, and the average insolvency put per dollar of liabilities is 0.15%. There is significant variation in the put value among the firms in the sample, from near zero to 10.75 percent of liabilities.

The average book and market value asset-to-liability ratios are 1.38 and 1.53, respectively. The market value asset-to-liability ratio, MVALNT, ranges from 0.99 to 3.54 while the corresponding range for the book value asset-to-liability ratio, BVLAR, is 1.00 to 2.28. This suggests that investors are, as predicted, re-valuing the assets and liabilities on the balance-sheets at market values and also valuing other items not recorded on the balance sheet. It is also interesting to note that the market derived asset-to-liability ratios reported here for insurers are much higher than the ratios reported for commercial banks. Cordell and King (1995) report market derived asset-to-liability ratios averaging about 1.06 for their sample of commercial banks and 1.03 for their sample of savings and loans. Thus, insurers appear to compensate for their higher volatilities by holding more capital, a finding consistent with Cummins and Sommer (1996).

#### *Price Variability Across Insurers*

To test the hypothesis that the price of insurance is inversely related to the riskiness of the firm (Hypothesis 1), the following regression is estimated separately for the short-tail lines and the long-tail lines:

$$\begin{aligned} PRY_{jt} = & \alpha + \beta_1 LPUTNT_{jt} + \beta_2 LPUTNT2_{jt} \\ & + \beta_3 LIABGROY_{jt} + \beta_4 COHERF_{jt} + \varepsilon_{jt} \end{aligned} \quad (26)$$

where  $PRY_{jt}$  = the premium-to-loss ratio for company  $j$  in year  $t$  for either the short-tail ( $Y = S$ ) or long-tail ( $Y = L$ ) line,

<sup>29</sup>The hypothesis tests also were conducted using weekly data on insurer equity returns. We only report figures based upon daily data to conserve space and because the shorter time interval is preferable when estimating instantaneous volatility parameters. The weekly results are similar and are available from the authors on request.

- $LPUTNT_{jt}$  = ratio of the insolvency put (based on daily data after adjusting for non-synchronous trading) to total liabilities for company  $j$  in year  $t$ ,  
 $LPUTNT2_{jt}$  =  $LPUTNT_{jt}^2$ ,  
 $LIABGROY_{jt}$  = the liability growth rate for either the short-tail ( $Y = S$ ) or long-tail ( $Y = L$ ) line, and  
 $COHERF_{jt}$  = Herfindahl index for the concentration of premiums written among members of insurance groups =  $\sum_k (PW_{jkt}/PW_{jt})^2$ , where  $PW_{jkt}$  = premiums written for group member  $k$  of group  $j$  in year  $t$ , and  $PW_{jt}$  = total premiums written of group  $j$  in year  $t$ .<sup>30</sup>

The regressions were run separately for long-tail lines and short-tail lines because the pricing relationships may vary somewhat between the two major classes of business. Equation (26) is also estimated with the economic premium-to-loss ratio as the dependent variable.

The equations were estimated using our pooled time-series, cross-section sample of traded insurers for the period 1988 to 1992. Because the error structure is likely to differ among the companies in our sample, panel data methods were used to estimate the models. Both fixed and random effects versions of equation (26) were estimated, with the fixed effects version including both year and company effects. The models were estimated for the entire sample of companies and for the complete panel of companies available for the entire sample period. The results are robust to the choice of sample. Accordingly, we report only one set of results, based on the entire sample.

The risk measure used in these regressions is the estimate of the firm's insolvency put divided by the total liabilities of the firm. This is the theoretically most appropriate variable because it captures all of the factors that determine the overall riskiness of the firm.<sup>31</sup> The expected sign of this variable is negative. The reported regression results are based on the variable  $LPUTNT$ , which incorporates the daily estimates of the implied volatilities of the firms, adjusted for non-synchronous trading (SIGMANT). Regression results using other estimates of the insolvency put were similar. We also included the  $LPUTNT$  variable squared,  $LPUTNT2$ , to control for the possibility of a non-linear relationship between the put variable and the premium-to-loss ratio. We also report regressions where  $LPUTNT2$  is excluded from the model.

In addition to the insolvency put, differences in the liability growth rates across companies are predicted to affect the price of insurance. As reported above, we estimate growth rates by line based on industry-wide data in order to smooth out idiosyncratic fluctuations among firms that are unlikely to be incorporated in expectations. We vary the growth rates across firms by computing firm-specific growth rates equal to the loss reserve weighted average of the industry-wide growth rates by line. The firm-specific growth rates

<sup>30</sup>This definition of the intra-group concentration index is similar to the one used in Sommer (1996). In addition to this definition, we also measured the intra-group concentration index using the liability holdings and the surplus levels of the group members. Because the results for the three measures of concentration are very similar, we report only the results based on premiums written for consistency with the prior literature.

<sup>31</sup>Regressions were also run where a volatility measure of the firm and the market-value leverage measure were substituted for the insolvency put variable and similar results were obtained.

thus reflect each firm's mix of business across lines. Two weighted average growth rates are estimated for each company: LIABGROL for long-tailed liabilities and LIABGROS for short-tailed liabilities. The expected signs of these variables are ambiguous (see expressions (19) and (20)). However, for most reasonable parameter values we expect the estimated coefficients to be negative.

Finally, we include the Herfindahl index measuring the concentration of premiums written among members of insurance groups. Higher values of the index imply less dispersion of premiums written among group members, with a value of 1 implying that all premiums written are concentrated in a single company (i.e., the company consists of a single corporate entity with no subsidiaries). This variable is used to test the "corporate veil" hypothesis (Hypothesis 3), i.e., other things equal, default risk is less if business of the insurer is highly concentrated rather than widely dispersed among different corporate entities within the group. Thus, the expected sign of this variable is positive.

The regression results for the fixed effects specification based on equation (26) are reported in Table 2. The results provide support for Hypotheses 1 and 3. In the regressions for the long-tail lines, where the squared value of the insolvency put variable is excluded, the coefficient of the insolvency put risk variable (LPUTNT) is negative, as predicted by the model, and statistically significant at the 1 percent level or better. Inclusion of the squared value of put variable (LPUTNT2) reduces the significance of the put variable (LPUTNT) and reduces the adjusted- $R^2$ . In addition, the coefficient of the squared put variable is not statistically significant. Thus, the quadratic specification does not seem appropriate for the long-tail lines; and on balance the results are supportive of Hypothesis 1, i.e., that price is inversely related to default risk. Likewise, the coefficient of the company liability Herfindahl index is positive and significant in all long-tail regressions, supporting Hypothesis 3 and suggesting the presence of a market price penalty for dispersion of business across members of insurance groups. The liability growth rate is negative and significant in all of the long-tail regressions.

The results are similar for the short-tail lines, with the exception of the insolvency put variable. In the versions of the regressions that exclude the squared value of the put variable, the coefficient of the insolvency put variable is positive, contrary to expectations, but not statistically significant. When the squared value of the put variable is included in the equations to allow for non-linearity, the coefficient of the put variable is negative as expected and statistically significant at the 5 percent level or better. In addition, the squared value of the put variable has a statistically significant positive coefficient. Thus, we find evidence of a non-linear, inverse relationship between price and the insolvency put for the short-tail lines. Although the quadratic specification introduces the possibility that the net effect of the put value could be positive for some observations, only two of the 315 observations in the sample have values of the put variable in the range where this would be true. I.e., the partial derivative of the estimated quadratic specified regression equation with respect to the insolvency put is positive for only two observations. Thus, the quadratic specification provides further support for Hypothesis 1. Hypothesis 3 is also supported for the short-tail lines.

**Table 2**  
Inter-Company Price Tests Controlling for Year and Company Fixed Effects

Long-tailed Lines									
Dependent									
Variable	Intercept		LPUTNT		LPUTNT2		LIABGROL	COHERF	Adj-R <sup>2</sup>
PRL	0.9508 ***		-2.1998 ***		-		-0.8448 **	0.2776 ***	0.5927
	(0.0767)		(0.8484)				(0.3890)	(0.0647)	
EPRL	1.2063 ***		-2.5478 ***		-		-1.0026 **	0.2815 ***	0.5996
	(0.0949)		(1.0519)				(0.4824)	(0.0802)	
PRL	0.9484 ***		-4.5814 *		28.2752		-0.8437 **	0.2812 ***	0.5921
	(0.0766)		(3.3702)		(35.2796)		(0.3893)	(0.0649)	
EPRL	1.2037 ***		-5.2471		29.1977		-1.0014 **	0.2852 ***	0.5987
	(0.0951)		(4.1806)		(43.7627)		(0.4829)	(0.0805)	
Short-tailed Lines									
Dependent									
Variable	Intercept		LPUTNT		LPUTNT2		LIABGROS	COHERF	Adj-R <sup>2</sup>
PRS	1.1039 ***		2.9287		-		-1.1152 **	0.3961 ***	0.3520
	(0.1218)		(2.1183)				(0.6048)	(0.0163)	
EPRS	1.2013 ***		3.1011		-		-1.1553 **	0.3910 **	0.3554
	(0.1276)		(2.2177)				(0.6332)	(0.1705)	
PRS	1.0844 ***		-18.9188 **		236.3335 ***		-1.1040 **	0.4259 ***	0.3680
	(0.1205)		(8.3394)		(87.3264)		(0.5973)	(0.1612)	
EPRS	1.1811 ***		-19.5336 **		244.8751 ***		-1.1437 **	0.4219 ***	0.3709
	(0.1262)		(8.7333)		(91.4520)		(0.6255)	(0.1688)	
*** - significant at 1 % level; ** - significant at 5 % level; * - significant at 10 % level; one-tailed t-test									
Standard errors reported in parentheses									
Number of observations - 315									
PRL (EPRL) - Long tailed premium (economic premium) ratio net of underwriting expenses									
PRS (EPRS) - Short-tailed premium (economic premium) ratio net of underwriting expenses									
LPUTNT - Insolvency put estimated from non-synchronous adjusted daily data									
LPUTNT2 - LPUTNT <sup>2</sup>									
LIABGROL (LIABGROS) - Long-tailed (Short-tailed) liability growth rate									
COHERF - Herfindahl measuring concentration of premiums among members of the insurance group									

The random effects versions of the model are reported in Table 3. We report Hausman chi-square statistics to test the null hypothesis that random effects are appropriate against the alternative hypothesis that the model is characterized by fixed effects. These tests do not reject the null hypothesis that random effects are appropriate except at the 10 percent level for the long-tailed runs where the dependent variable is the long-tailed premium-to-loss ratio, LPRE. Thus, the random effects models are, in general, preferred to the fixed effects models.

The random effects results are similar to the fixed effects results except that the coefficient of the squared put value (LPUTNT2) is positive and statistically significant in

**Table 3**  
Inter-Company Price Tests Controlling for Year and Company Random Effects

<i>Long-tailed Lines</i>							Hausman $\chi^2$ Statistic
Dependent Variable	Intercept	LPUTNT	LPUTNT2	LIABGROL	COHERF		
PRL	0.8773 *** (0.0557)	-2.4696 *** (1.0082)	-	-0.7992 ** (0.3888)	0.2128 *** (0.0492)		6.860 *
EPRL	1.0730 *** (0.0745)	-2.8325 ** (1.2813)	-	-1.0545 ** (0.5065)	0.2301 *** (0.0618)		6.064
PRL	0.8702 *** (0.0549)	-9.7238 *** (3.8678)	79.2970 ** (40.9102)	-0.7512 ** (0.3852)	0.2226 *** (0.0492)		7.932 *
EPRL	1.0661 *** (0.0739)	-10.1120 ** (4.9143)	79.5640 * (51.9603)	-1.0083 ** (0.5043)	0.2399 *** (0.0620)		6.692
<i>Short-tailed Lines</i>							Hausman $\chi^2$ Statistic
Dependent Variable	Intercept	LPUTNT	LPUTNT2	LIABGROS	COHERF		
PRS	1.0189 *** (0.0830)	2.1237 (2.2441)	-	-0.9094 ** (0.4040)	0.3301 *** (0.1063)		2.132
EPRS	1.0748 *** (0.0891)	2.1922 (2.3438)	-	-0.9960 *** (0.4226)	0.3513 *** (0.1110)		2.191
PRS	1.0135 *** (0.0821)	-21.9800 *** (8.4902)	264.3500 *** (89.9204)	-0.9842 *** (0.3995)	0.3611 *** (0.1056)		2.448
EPRS	1.0691 *** (0.0879)	-22.6340 *** (8.8668)	272.2300 *** (93.9001)	-1.0716 *** (0.4180)	0.3834 *** (0.1103)		2.561

\*\*\* - significant at 1 % level; \*\* - significant at 5 % level; \* - significant at 10 % level; one-tailed t-test

Standard errors reported in parentheses

Number of observations - 315

PRL (EPRL) - Long tailed premium (economic premium) ratio net of underwriting expenses

PRS (EPRS) - Short-tailed premium (economic premium) ratio net of underwriting expenses

LPUTNT - Insolvency put estimated from non-synchronous adjusted daily data

LPUTNT2 - LPUTNT<sup>2</sup>

LIABGROL (LIABGROS) - Long-tailed (Short-tailed) liability growth rate

COHERF - Herfindahl measuring concentration of premiums among members of the insurance group

the long-tail runs, suggesting the presence of a non-linear relationship for the long-tail lines. In the quadratic specifications, all companies have values of the put variable in the range where the partial of the price with respect to the put variable is negative. Thus, both the quadratic and non-quadratic specifications support Hypothesis 1 for the long-tail lines.

As in the fixed effects regressions, the quadratic random effects regressions support Hypothesis 1 for the short-tail lines, while the regressions including only the first-order put

variable do not support the hypothesis. However, in view of the strong evidence of a non-linear relationship for the short-tail lines and the fact that none of the observations are in the range where the quadratic specifications would suggest a positive relationship between price and the put, we conclude that the short-tail random effects regressions provide further support for Hypothesis 1. The random effects regressions also consistently support Hypothesis 3 for both the long and short-tail lines, i.e. dispersion of business among subsidiaries is inversely related to price.

The results with respect to Hypothesis 1 support our argument that guaranty fund protection is less than complete. However, the existence of guaranty funds may provide an explanation for the more pronounced non-linearity in the short-tail lines than in the long-tail lines, which suggests that the marginal effect of increases in the insolvency put diminish more rapidly for the short-tail lines. Short-tail claims (such as automobile property damage claims) tend to be smaller on average than long-tail claims (such as bodily injury liability claims), so that claimants are more likely to reach the guaranty fund claim cap for long-tail than for short-tail claims. The insurer's claims adjustment services also are more important in the long-tail liability lines because they involve providing a legal defense. In general, financially sound insurers are likely to provide higher quality legal defense services than the individual can acquire on his or her own by shopping the market for lawyers, because insurers tend to have superior information about the skills of defense attorneys. Finally, because long-tail claims settle more slowly and, for liability coverages, may not be filed until long after the policy period has ended (the so-called incurred but not reported claims), the long-tail claimant or policyholder runs more risk of having a claim denied due to the "late-filing" provisions of guaranty fund laws (see National Association of Insurance Commissioners 1993). Thus, we expect the relationship between the put and price to be stronger in the long-tail than in the short-tail lines.<sup>32</sup> The strong support for Hypothesis 3 provided by our empirical analysis also is noteworthy. Even though it is well-known that a parent corporation has the option to allow a subsidiary to fail (e.g., Easterbrook and Fischel 1985), we are aware of no prior research providing an empirical link between this option and the cost of debt capital. Our results suggest that the option has significant value that is recognized in the market for insurance.

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<sup>32</sup>A counter-argument is that purchasers of third party coverages such as liability insurance (which are predominantly long-tail lines) are likely to be less concerned about solvency than purchasers of first party coverages such as homeowners insurance (which is predominantly a short-tail line) because the benefit payments from third party coverages are not made to the policyholder but rather to third parties. This argument implies that the insolvency put option should have a greater effect on short-tail than on long-tail lines. The reasoning is that the insured is likely to be more concerned about recovering his/her own losses rather than providing payments to third parties. We consider this argument less persuasive than the one we advance in the text because the policyholder remains liable for any part of a liability judgment not paid by the guaranty fund and, moreover, must pay legal defense costs that would have been absorbed by the insurer if the insurer had remained solvent and defended the case. Our empirical results are consistent with the view that insurance buyers are at least as concerned about solvency in third party lines as in first party lines.

### *Intra-Insurer Cross-Line Price Variability*

We next investigate the hypothesis that premium-to-loss ratios are equal across lines of business within the same insurer after accounting for overall firm risk and differences in line-specific liability growth rates (Hypothesis 2). The dependent variable for this test is the natural logarithm of the ratio of the economic premium-to-loss ratio for the short-tail lines to the economic premium-to-loss ratio for the long-tail lines. The economic premium-to-loss ratio is used rather than the premium-to-loss ratio to control for the loss payment timing differences between the lines of business.

We first conduct a simple t-test of the null hypothesis that the logarithm of the short-tail to long-tail economic premium-to-loss ratio is equal to zero (i.e., that the ratio is equal to 1) by computing the mean and standard deviation of the ratio across the observations in our sample.<sup>33</sup> Since these tests do not account for differences between line liability growth rates and do not explicitly control for firm risk, we expect to reject the null hypothesis. To test the hypothesis controlling for these factors, we estimate the following regression:

$$LRATIO_{jt} = \ln\left(\frac{EPRS_{jt}}{EPRL_{jt}}\right) = \alpha + \beta_1 LIABGROS_{jt} + \beta_2 LIABGROL_{jt} + \beta_3 LPUTNT_{jt} + \beta_4 COHERF_{jt} + \varepsilon_{jt} \quad (27)$$

where  $EPRS_{jt}$ ,  $EPRL_{jt}$  = the economic premium-to-loss ratio for company  $j$  in year  $t$ , for the short and long-tail lines, respectively. Equation (27) controls for both firm-specific default risk and short-tail and long-tail liability growth rates. The null hypothesis is that the intercept is equal to zero. Failure to reject the null hypothesis would provide support for Hypothesis 2, that prices across lines within a firm depend only on overall firm risk, not the risk of the individual lines.

Equation (27) was estimated using both fixed and random effects. Because some important lines of insurance such as workers' compensation and private passenger auto insurance are subject to price regulation in many states, we conduct the tests both including and excluding data from these two heavily regulated lines, based on the rationale that price regulation can prevent prices from reaching their competitive equilibrium levels.

The results for all lines of insurance are presented in Table 4. As expected the null hypothesis that the dependent variable is equal to zero is rejected based on the simple t-test. In the fixed effects regression, the null hypothesis that the intercept is equal to zero is rejected at the 10 percent level. However, we cannot reject the null hypothesis based on the random effects regression, and the Hausman test (Chi-square statistic) fails to reject the hypothesis that the random effects specification is superior to the fixed effects specification. Thus, the random effects specification is preferred; and, on balance, the results support Hypothesis 2. The fact that the intercept is not significantly different from zero is the key here, implying that the ratio of the short-tail to long-tail economic premium ratios is not

<sup>33</sup>As in the tests reported above, we include all valid observations, i.e., we do not require firms to be present in all years of the sample period. Tests based on the subset of firms with data for all years yielded similar results.

significantly different from one after controlling for the other factors represented by the independent variables in the regression.

**Table 4**  
Intra-Company Price Tests - All Lines of Business

<i>Simple T-test</i>						
Variable	Mean	St. Dev.	T-Statistic	Prob  t  > 0		
LRATIO	0.0748	0.3824	3.470	0.1%		

  

<i>Controlling for Year and Company Fixed Effects</i>						
Dependent						
Variable	Intercept	LIABGROS	LIABGROL	LPUTNT	COHERF	Adj -R <sup>2</sup>
LRATIO	0.2977 *	-1.2540 **	-0.7994	4.4216 **	-0.0205	0.4228
	(0.1634)	(0.5060)	(0.8163)	(1.7694)	(0.1356)	

  

<i>Controlling for Year and Company Random Effects</i>						
Dependent						Hausman
Variable	Intercept	LIABGROS	LIABGROL	LPUTNT	COHERF	$\chi^2$ Statistic
LRATIO	0.0960	-1.4093 ***	0.3757	4.0539 **	0.0185	3.6448
	(0.0858)	(0.3421)	(0.5932)	(1.8690)	(0.0904)	

\*\*\* - significant at 1 % level; \*\* - significant at 5 % level; \* - significant at 10 % level; two-tailed t-test

Standard errors reported in parentheses

Number of observations - 315 all lines

LRATIO - Natural logarithm short-tailed economic premium ratio net of underwriting expenses to the long-tailed economic premium ratio net of underwriting expenses

LIABGROL (LIABGROS) - Long-tailed (Short-tailed) liability growth rate

LPUTNT - Insolvency put estimated from non-synchronous adjusted daily data

COHERF - Herfindahl measuring concentration of premiums among members of the insurance group

As suggested above, the all lines results may be distorted by price regulation. The primary effect of regulation is on private passenger auto insurance and workers' compensation insurance.<sup>34</sup> Thus, to further investigate Hypothesis 2, the relevant variables

<sup>34</sup>Both lines of business are characterized by large involuntary markets in many states and there are often threats of private insurers abandoning the market due to inadequate rates. Grabowski, Viscusi and Evans (1989) found that rate regulation held premium-to-loss ratios for automobile liability insurance below competitive levels during the mid-1980's. Evidence on the effects of regulation on workers' compensation insurance prices is provided by Carroll (1993).

were recalculated after removing private passenger automobile and workers' compensation insurance.<sup>35</sup> The results are shown in Table 5.

**Table 5**  
Intra-Company Price Tests - Unregulated Lines of Business Only

<i>Simple T-Test</i>						
Variable	Mean	St. Dev.	T-Statistic	Prob	ltn > 0	
LRATIO	0.0633	0.4422	2.475	1.4%		

  

<i>Controlling for Year and Company Fixed Effects</i>						
Dependent						
Variable	Intercept	LIABGROS	LIABGROL	LPUTNT	COHERF	Adj -R <sup>2</sup>
LRATIO	0.1838 (0.2164)	-1.0127 (0.6436)	0.1279 (0.1051)	5.7856 ** (2.7191)	-0.0511 (0.1984)	0.3168

  

<i>Controlling for Year and Company Random Effects</i>						
Dependent						Hausman
Variable	Intercept	LIABGROS	LIABGROL	LPUTNT	COHERF	$\chi^2$ Statistic
LRATIO	0.1331 (0.1031)	-0.6122 (0.4430)	0.3541 (0.6611)	5.1628 * (2.7720)	-0.1294 (0.1172)	3.9796

\*\*\* - significant at 1 % level; \*\* - significant at 5 % level; \* - significant at 10 % level; two-tailed t-test  
Standard errors reported in parantheses  
Number of observations - 299 unregulated lines  
LRATIO - Natural logarithm short-tailed economic premium ratio net of underwriting expenses to the long-tailed economic premium ratio net of underwriting expenses  
LIABGROL (LIABGROS) - Long-tailed (Short-tailed) liability growth rate  
LPUTNT - Insolvency put estimated from non-synchronous adjusted daily data  
COHERF - Herfindahl measuring concentration of premiums among members of the insurance group

As in the case of the all lines results, the null hypothesis that the short and long-tail economic premium-to-loss ratios are equal is rejected based on a simple t-test. However, we cannot reject the null hypothesis based on either the fixed or the random effects regressions. Thus, after controlling for regulation, we find clear support for Hypothesis 2. Besides suggesting that regulation distorts prices, our findings imply that pricing methods based on

<sup>35</sup> Another possibility would be to eliminate private passenger auto and workers' compensation insurance experience only for regulated states rather than for all states or to exclude these lines only for stringently regulated states. Because good measures of regulatory stringency do not exist for all years of our sample period, we concluded that it made more sense to eliminate these two lines of business entirely for this set of tests. However, it would be useful for future research on this topic to explore the effect other regulatory exclusion rules on tests of Hypothesis 2.

allocations of equity by line of insurance are not consistent with insurance prices observed in the market place.<sup>36</sup>

## CONCLUSION

This paper develops a financial pricing model for multiple line insurers subject to default risk. It overcomes the principal limitations of prior financial pricing models for insurance, which either apply to mono-line insurers or require the allocation of equity capital by line. Using an option pricing framework, we show that the informationally-efficient, competitive market price of insurance for a given line of business depends on the overall risk of the firm rather than the risk of the individual line being priced. This rather remarkable result is due to the fact that it is not the equity of the insurer but rather the expected cost of insolvency that should be allocated to the various divisions of the firm.

The model yields two primary empirical predictions: (1) the price of insurance should be inversely related to firm default risk, and (2) insurance prices across lines of business for a given insurer should be equal after controlling for default risk and line-specific liability growth rates. In addition, we hypothesize that the price of insurance should be inversely related to the dispersion of business among the subsidiaries of an insurance group because the “corporate veil” doctrine provides a valuable default option to the owners of an insurance group by preventing (in most cases) claimants against an insolvent subsidiary from reaching the assets of other subsidiaries or the parent corporation.

The empirical tests support the predictions of the model. In tests of price differences across insurers, it is shown that the price of insurance is inversely related to the riskiness of the firm, providing support for Hypothesis 1 and for the argument that guaranty fund protection is less than complete. This inverse relationship is stronger for long-tail lines of business than for short-tail lines, suggesting that the default premium increases the longer the payout tail. Line-specific growth rates are shown to have a statistically significant effect on the price of insurance, consistent with the theoretical model. We also provide evidence that the premium-to-loss ratio is inversely related to the dispersion of business among subsidiaries suggesting that the group’s option to allow a subsidiary to fail has significant value that is recognized in the market for insurance. We also examine price differences across lines of business within the same insurance company. Empirical support is provided for the hypothesis that the economic premium-to-loss ratios for a given insurer are equal

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<sup>36</sup>To provide another general indication of the reasonableness of the model, we also tested the performance of the model in predicting actual premium levels and economic premium-to-loss ratios. We use the premium equation (equation (14)) to predict aggregate pure premium levels by line and compare our forecasts with observed premiums net of expenses and policyholder dividends. To predict economic premium-to-loss ratios, we use equation (25) with our pure premium forecasts in the numerator and the discounted starting value of losses in the denominator. The predicted ratios are compared with actual economic premium-to-loss ratios in the following period. To measure predictive accuracy, we use Theil’s U statistic (see Theil 1966). The tests reveal that the model is highly accurate in predicting both aggregate premiums by line and economic premium-to-loss ratios. A decomposition of U (see Theil 1966) suggests that predictive performance is especially high for the long-tail lines, reinforcing the finding that long-tail lines are more responsive to default risk than short-tail lines. Details of the Theil tests are available from the authors.

across lines of business, after controlling for line-specific liability growth rates. The empirical evidence is broadly consistent with the view that insurance markets are informationally efficient and competitive and that price regulation has a distorting effect on the relative prices among lines of insurance. This provides further support for the argument that regulation is likely to have adverse effects on resource allocation and the quality and availability of insurance. The results also suggest that there is likely to be a market reward for the development and adoption of improved risk management techniques that enable insurers to efficiently reduce their default risk. This is an important message at a time when insurers are increasingly exploring innovative financial risk management techniques such as securitization, derivatives, and advanced asset-liability management methods.

One important avenue for future research would be to evaluate the pricing problem in a multi-period setting. The present model implicitly assumes that total incurred losses are known with certainty at the end of the policy period, but this may not be realistic for long-tail lines. A second area for further research will be to incorporate taxes into this model as taxes represent a significant cost of providing coverage. Finally, extending the model to include a catastrophic (jump) risk component could improve the ability of the model to predict prices for short-tail property lines that are subject to natural hazards such as hurricanes and earthquakes.

## APPENDIX 1

This appendix develops a financial pricing model which can be used to value the options of a two-line insurance company. Extension of the analysis to  $n$  lines of business is straight forward.

Assume there are two time periods, time 0 and time 1. The insurance company consists of two lines of business and equity holders. At time 0 premiums of  $P_i$  are collected from policyholders for line  $i$ , where  $i=1,2$ . The equity holders of the firm contribute surplus of  $G$ . Let  $A(0)=P_1+P_2+G$  be the market value of the assets of the company at time 0. Because of imperfect contracts, the premiums and the surplus are all paid at time 0 to avoid the possibility of nonpayment after the losses have been realized at time 1. In return for premiums, the insurance company agrees to underwrite the expected liability payments for each line of business,  $L_i$ .

The premiums for each line of business and the surplus will be invested. Assume the market value of the premiums, surplus and liabilities evolve according to the following stochastic processes:

$$dP_i = \mu_{P_i} P_i dt + \sigma_{P_i} P_i dz_{P_i} \quad (A1)$$

$$dG = \mu_G G dt + \sigma_G G dz_G \quad (A2)$$

where  $P_i, G, L_i$  = invested premiums, invested surplus, and liabilities for line  $i$ , respectively,  
 $\mu_{P_i}, \mu_G, \mu_{L_i}$  = instantaneous drift on invested premiums, surplus, and liabilities for line  $i$ ,

$$dL_i = \mu_{L_i} L_i dt + \sigma_{L_i} L_i dz_{L_i} \quad (A3)$$

$\sigma_{P_i}, \sigma_G, \sigma_{L_i}$  = instantaneous standard deviation of invested premiums, invested surplus, and liabilities for line  $i$ , and

$dz_{P_i}, dz_G, dz_{L_i}$  = standard diffusion process (Wiener process).

The instantaneous correlation coefficients between the diffusion processes are as follows:

$dz_{P_i} dz_G = \rho_{P_i G} dt$  for  $i=1,2$ ,  $dz_{P_i} dz_{P_j} = \rho_{P_i P_j} dt$  for  $i \neq j$ ,  $dz_{L_i} dz_{P_j} = \rho_{L_i P_j} dt$  for  $i=1,2$  and  $j=1,2$ ,  $dz_{L_i} dz_G = \rho_{L_i G} dt$  for  $i=1,2$ ,  $dz_{L_i} dz_{L_j} = \rho_{L_i L_j} dt$  for  $i \neq j$ .

Both assets and liabilities are assumed to be priced according to an intertemporal asset pricing model, such as the intertemporal capital asset pricing model (ICAPM). The ICAPM implies the following return relationships:

$$\mu_j = r_f + \pi_j, \text{ for } j = P_1, P_2, G$$

$$\mu_{L_i} = r_{L_i} + \pi_{L_i}, \text{ for liability classes } i=1,2$$

where  $r_f$  = risk-free interest rate,

$r_{L_i}$  = inflation rate in liability class  $i$ , and

$\pi_i$  = the market risk premium for asset  $i = P_1, P_2, G, L_1, L_2$ .

Since we have assumed that the invested assets and liabilities are priced according to the ICAPM, the risk premium,  $\pi_i$ , would be

$$\pi_i = \rho_{iM} (\sigma_i / \sigma_M) [\mu_M - r_f]$$

where  $\mu_M, \sigma_M$  are the drift and diffusion parameters of the market portfolio and  $\rho_{iM}$  is the correlation coefficient between the Brownian motion process for asset or liability  $i$  and the market portfolio.

The value of any divisional option, either the divisional call option or the divisional put option, can be written as  $H_i(P_i, L_i, \tau)$  where  $\tau$  is the time to expiration of the option. Differentiating  $H_i$  using Ito's lemma and invoking the ICAPM pricing relationships for the premiums and liabilities yields

$$\begin{aligned}
 Hr_f = & r_f H_{P_i} P_i + r_{L_i} H_{L_i} L_i - H_\tau + \frac{1}{2} \sigma_{P_i}^2 H_{P_i P_i} P_i^2 \\
 & + H_{P_i L_i} P_i L_i \rho_{P_i L_i} \sigma_{P_i} \sigma_{L_i} + \frac{1}{2} \sigma_{L_i}^2 H_{L_i L_i} L_i^2
 \end{aligned}
 \tag{A4}$$

The risk, i.e. any term multiplied by a  $dz_j$  term, and the market risk parameters, i.e., the  $\pi_j$  terms, have been eliminated by using the ICAPM and taking expectations. It is also possible to do this by using a hedging argument, such as the one used by Fischer (1978). However, this assumes that the appropriate hedging securities are available.

The next step is to use the homogeneity property of the options model to change variables so that the model is expressed in terms of the premium-to-liability ratio,  $x_i = P_i/L_i$ , and the option value-to-liability ratio  $h_i = H_i/L_i$ . The result is the following differential equation:

$$\begin{aligned}
 (r_f - r_{L_i}) h^i = & (r_f - r_{L_i}) x_i h^i_{x_i x_i} + \\
 & \frac{1}{2} (\sigma_{P_i}^2 + \sigma_{L_i}^2 - 2 \sigma_{P_i} \sigma_{L_i} \rho_{P_i L_i}) h^i_{x_i x_i} x_i^2 - h^i_\tau
 \end{aligned}
 \tag{A5}$$

where  $r_f$  = risk-free rate of interest

$r_{L_i}$  = inflation component of the instantaneous liability drift term,

$h^i = H_i(P_i, L_i, \tau)/L_i$ , and

$x_i = P_i/L_i$ .

Equation (A5) is the standard Black-Scholes differential equation, where the optioned asset is the premium-to-liability ratio for line  $i$ . To obtain the value of any specific claim on a division of the firm one would solve (A.5) subject to the appropriate boundary conditions. For example, the value of the option held by the equity holders of the firm which entitles them to the residual value of the division after all claims have been paid can be modeled as a call option. The boundary conditions for this option are  $C_i(0, L_i, \tau) = 0$ , and  $C_i(P_i, L_i, \tau) = \text{MAX}(P_i - L_i, 0)$ .

The process to find the value of any contingent claim on the entire firm is very similar to the methodology used to determine the value of the divisional options. The value of an option on the entire two-line insurance company can be written as  $H(P_1, P_2, G, L_1, L_2, \tau)$ . Differentiating  $H$  using Ito's lemma and invoking the ICAPM pricing relationships yields

$$\begin{aligned}
 Hr_f = & r_f H_{P_1} P_1 + r_f H_{P_2} P_2 + r_f H_G G + \\
 & r_{L_1} H_{L_1} L_1 + r_{L_2} H_{L_2} L_2 - H_\tau + \frac{1}{2} A V A'
 \end{aligned}
 \tag{A6}$$

where  $A$  is a vector equal to

$$A = (P_1 \ P_2 \ G \ L_1 \ L_2) \quad (A7)$$

and matrix  $V$  is equal to

$$V = \begin{pmatrix} H_{P_1 P_1} \sigma_{P_1}^2 & H_{P_1 P_2} \sigma_{P_1 P_2} & H_{P_1 G} \sigma_{P_1 G} & H_{P_1 L_1} \sigma_{P_1 L_1} & H_{P_1 L_2} \sigma_{P_1 L_2} \\ H_{P_2 P_1} \sigma_{P_2 P_1} & H_{P_2 P_2} \sigma_{P_2}^2 & H_{P_2 G} \sigma_{P_2 G} & H_{P_2 L_1} \sigma_{P_2 L_1} & H_{P_2 L_2} \sigma_{P_2 L_2} \\ H_{G P_1} \sigma_{G P_1} & H_{G P_2} \sigma_{G P_2} & H_{G G} \sigma_G^2 & H_{G L_1} \sigma_{G L_1} & H_{G L_2} \sigma_{G L_2} \\ H_{L_1 P_1} \sigma_{L_1 P_1} & H_{L_1 P_2} \sigma_{L_1 P_2} & H_{L_1 G} \sigma_{L_1 G} & H_{L_1 L_1} \sigma_{L_1}^2 & H_{L_1 L_2} \sigma_{L_1 L_2} \\ H_{L_2 P_1} \sigma_{L_2 P_1} & H_{L_2 P_2} \sigma_{L_2 P_2} & H_{L_2 G} \sigma_{L_2 G} & H_{L_2 L_1} \sigma_{L_2 L_1} & H_{L_2 L_2} \sigma_{L_2}^2 \end{pmatrix}. \quad (A8)$$

Note, the risk and drift parameters have again been eliminated by using the ICAPM pricing relationships and taking expectations. The next step is to use the homogeneity property of the options model again. This time we want to express the model in terms of the asset-to-liability ratio  $x=A/L$  where  $A=P_1+P_2+G$  and  $L=L_1+L_2$ , and in terms of the option-to-liability ratio  $h=H/L$  and the liability and asset proportions, i.e.  $w_{L_i}=L_i/L$  and  $w_{P_i}=P_i/A$  and  $w_G=G/A$ . Note, this requires us to make the assumption that the sum of lognormally distributed random variables are lognormally distributed, e.g., that  $L_1+L_2$  can be approximated by a lognormal diffusion process. The assumption about the additivity of lognormals is routinely used in the discrete time option pricing literature (e.g., Stapleton and Subramanyam, 1984). The result is the following differential equation

$$r_x h = r_x x h_x + \frac{1}{2} x^2 h_{xx} \sigma_x^2 - h_\tau \quad (A9)$$

where  $r_x = r_f - w_{L_1} r_{L_1} - w_{L_2} r_{L_2}$

$$\sigma_x^2 = W \Sigma W'$$

$$A = P_1 + P_2 + G$$

$$L = L_1 + L_2$$

$$h = H(A, L, \tau)/L$$

$$x = A/L$$

$$w_{P_i} = P_i/A, \text{ for } i = 1, 2$$

$$w_G = G/A$$

$$w_{L_i} = L_i/L, \text{ for } i = 1, 2$$

$$W = (w_{P_1} \ w_{P_2} \ w_G \ w_{L_1} \ w_{L_2}), \text{ and}$$

$$\Sigma = \begin{pmatrix} \sigma_{P_1}^2 & \sigma_{P_1 P_2} & \sigma_{P_1 G} & \sigma_{P_1 L_1} & \sigma_{P_1 L_2} \\ \sigma_{P_2 P_1} & \sigma_{P_2}^2 & \sigma_{P_2 G} & \sigma_{P_2 L_1} & \sigma_{P_2 L_2} \\ \sigma_{G P_1} & \sigma_{G P_2} & \sigma_G^2 & \sigma_{G L_1} & \sigma_{G L_2} \\ \sigma_{L_1 P_1} & \sigma_{L_1 P_2} & \sigma_{L_1 G} & \sigma_{L_1}^2 & \sigma_{L_1 L_2} \\ \sigma_{L_2 P_1} & \sigma_{L_2 P_2} & \sigma_{L_2 G} & \sigma_{L_2 L_1} & \sigma_{L_2}^2 \end{pmatrix}.$$

Equation (A9) is the standard Black-Scholes differential equation, where the optioned asset is the asset-to-liability ratio of the entire firm,  $x$ .

## APPENDIX 2

### Publicly Traded Property-Liability Insurance Companies Included In the Sample

Aetna	Foremost Corp of America	Orion Capital Corp.
ALFA Corp	Fremont General	Phoenix RE Corp.
Allied Group Inc	Frontier Insurance	Piedmont Management
American Bankers Insurance	GEICO	Progressive Corp
American Indemnity	GAINSCO	RLI Corp
American International Group	General RE Corp	Reliance Corp
Argonaut Group Inc	Hartford Steam and Boiler	Riverside Group
AVEMCO Corp	Independent Ins. Group	Scor US Reinsurance
Baldwin & Lyons	Kemper Corp	SAFECO
Berkley (WR) Corp	Lawrence Insurance Group	St. Paul Companies
Berkshire Hathaway	Lincoln National	Seibels Bruce Group
Cigna	Merchants Group	Selective Insurance
CNA	Mercury General Corp.	State Auto Financial
Capitol Transamerica	Meridian Ins Group	Transamerica
Chubb	Milwaukee Ins Group	Travelers Corp
Cinncinnati Financial	Mobile America Corp.	20th Century Industries
Citation Insurance Group	Nac RE Corp	USFG
Citizens Security Group	Nymagic Corp	United Fire and Casualty
Condor Services Inc	National RE	United State Facilities
Continental Corp	National Security Co.	Unitrin Inc
Danielson Holding Co	Navigators Group	Victoria Financial
Donegal Insurance Group	North East Ins Co.	Walshire Assurance
EMC Insurance Group	Ohio Casualty	Zenith National Insurance
First Central Financial	Old Republic International	

### Line of Business Definitions

Short-tailed Lines	Long-tailed Lines
Fire	Farmowners Multiple Peril
Allied Lines	Homeowners Multiple Peril
Mortgage Guaranty	Commercial Multiple Peril

$$\begin{aligned}
h &= H(A, L, \tau)/L \\
x &= A/L \\
w_{P_i} &= P_i/A, \text{ for } i = 1, 2 \\
w_G &= G/A \\
w_{L_i} &= L_i/L, \text{ for } i = 1, 2 \\
W &= (w_{P_1} \ w_{P_2} \ w_G \ w_{L_1} \ w_{L_2}), \text{ and} \\
\Sigma &= \begin{pmatrix} \sigma_{P_1}^2 & \sigma_{P_1 P_2} & \sigma_{P_1 G} & \sigma_{P_1 L_1} & \sigma_{P_1 L_2} \\ \sigma_{P_2 P_1} & \sigma_{P_2}^2 & \sigma_{P_2 G} & \sigma_{P_2 L_1} & \sigma_{P_2 L_2} \\ \sigma_{G P_1} & \sigma_{G P_2} & \sigma_G^2 & \sigma_{G L_1} & \sigma_{G L_2} \\ \sigma_{L_1 P_1} & \sigma_{L_1 P_2} & \sigma_{L_1 G} & \sigma_{L_1}^2 & \sigma_{L_1 L_2} \\ \sigma_{L_2 P_1} & \sigma_{L_2 P_2} & \sigma_{L_2 G} & \sigma_{L_2 L_1} & \sigma_{L_2}^2 \end{pmatrix}.
\end{aligned}$$

Equation (A9) is the standard Black-Scholes differential equation, where the optioned asset is the asset-to-liability ratio of the entire firm,  $x$ .

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Inland Marine	Ocean Marine
Financial Guaranty	Medical Malpractice
Earthquake	International
Fidelity	Reinsurance
Surety	Workers' Compensation
Glass	Other Liability
Burglary and Theft	Products Liability
Credit	Aircraft
Automobile Physical Damage	Boiler and Machinery
	Automobile Liability

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