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Pricing and capital allocation in catastrophe insurance[☆]

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Abstract

This paper studies multi-line pricing and capital allocation by insurance companies when solvency matters to consumers, capital is costly to hold, and the average loss is uncertain. In this environment, product quality concerns lead firms to diversify across markets and charge high prices for risk that threatens company solvency, even if the risk is unrelated to other asset risk. Price differences across markets are traced to differences in capital required at the margin to maintain solvency. Finally, the paper shows that capital costs have significant effects on catastrophe insurance markets because of high marginal capital requirements. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Recent studies of the catastrophe reinsurance market find prices substantially in excess of expected losses (Froot, 1999; Froot and O'Connell, 1999). Yet catastrophe risk appears to be unrelated to financial asset returns and small relative to the capital market. Why are the prices so high? What is the cost to the insurer of providing this coverage? If the prices are driven by capital requirements, how much capital does an insurer "need" to underwrite catastrophe risk? The absence of definitive answers to these questions has bred a variety of opinions about the market's performance and the need for government intervention.

This paper studies the problem by developing a general model of multi-line insurance pricing and capital allocation by limited liability companies, incorporating three key assumptions. First, because there is uncertainty in the average loss, insurers may default. Second, it is costly for firms to hold capital. Third, the risk of insolvency matters to consumers.

In this environment, the insurer faces a cost–quality trade-off when choosing a level of capital holdings. The value of its promise to pay depends on its solvency, so it must balance the benefits that capital brings (greater security for policyholders) with the holding cost. Furthermore, the insurer has incentives to *economize* on capital through risk management and diversification. If the company reduces its risk, it can reduce capital and deliver the same level of security to policyholders at a lower cost. The motivation to reduce capital creates aversion to risk at the firm level. It will pay to avoid risk and charge to bear it, with the risk charge in a given market segment being determined by that segment's associated marginal capital requirement—the capital required to maintain company solvency at the margin as coverage is expanded in that segment. Price differentials across market segments are therefore explained by differences in marginal capital requirements. Segments with risk that threatens company solvency will have higher marginal capital requirements and higher prices due to implicit capital costs, even if that risk is unrelated to the broader securities markets.

Viewed from a different perspective, the risk penalties reflect the social aspects of insurance production. All policyholders in a given firm are "in the same boat" in the sense that they are potential claimants on the same set of assets. Since the overall quality of the company's product is at issue, the consequences of underwriting any individual contract cannot be evaluated in a vacuum. High-risk consumers generate negative effects on quality and must compensate the other policyholders for entry into the company. They must "pay their way" by paying for additional capital or by subsidizing the prices paid by other consumers. Because of these interdependencies among consumers, even competitive insurance prices will penalize risk and may lead to low participation rates in high-risk market segments.

Capital costs drive these results, and this paper examines their empirical importance. While capital costs are a relatively small part of production cost in the overall industry, they are significant in lines (such as catastrophe insurance) that use large amounts of supporting capital. As the amount of capital held increases, the frictional and tax costs associated with holding capital become a large part of the premium. In fact, in some capital-intensive lines of insurance, capital costs make up the majority of the premium.

This paper relates to the risk management literature, especially Froot and Stein (1998), Froot and O'Connell (1997), and Doherty (1991), who integrate risk management issues with standard asset pricing. It also relates to the capital allocation literature, such as Merton and Perold (1993) and Myers and Read (2001). Myers and Read develop a multi-line capital allocation rule for insurance, which is a special case of a general approach developed in this paper. It also builds on the insurance pricing literature,[1] especially papers incorporating default (e.g., Doherty and Garven, 1986; Cummins, 1988). By using consumer preferences as the motivation for risk management,[2] the model in this paper offers prices and capital allocations developed in the context of a competitive market-based solvency standard.

The rest of this paper is organized as follows. Section 2 shows how insurers choose prices and capital holdings when the cost of capital is unrelated to insurance risk. I study both monopoly and competitive pricing, with competitive pricing analyzed as the limiting case of monopoly pricing with perfectly elastic demand. Capital requirements by line of insurance are derived in a competitive setting. Section 3 derives the cost of capital and extends the theoretical results to the case in which insurance risk affects the cost of capital. Section 4 studies the impact of capital costs on insurance prices, revealing a significant impact in heavily capitalized lines of insurance. Section 5 concludes.

2. Pricing and capital allocation

This paper assumes that consumers care about the financial condition of insurers. Companies failed more frequently in earlier times, but default remains a threat even today. Property-casualty companies have failed at a rate of close to 1% per year over the past three decades (see A.M. Best Company, 1999), and life-health insurers have failed at similar rates in the recent past (A.M. Best Company, 1992). Even though guaranty funds are now in place in all states, policyholders still bear some of the burden of insolvency. Recoveries are capped, delayed, and subjected to additional deductibles. In addition, guaranty funds do not cover some classes of policyholders (notably, insurance companies). Testimony to policyholders' concern about solvency is found in market prices (see Sommer, 1996) and behavior.[3]

[1] For a recent review, see Cummins and Phillips (2000). Also relevant is literature stressing the importance of capital in pricing and equilibrium, such as Cagle and Harrington (1995), Winter (1994), and Gron (1994a,b).

[2] Research on the importance of consumer solvency preferences in guiding firm behavior and market equilibrium includes Doherty and Tinic (1981), Hoerger et al. (1990), Taylor (1994), Cummins and Danzon (1997), and Cummins and Sommer (1996).

[3] Policyholders of Executive Life and Mutual Benefit made "bank runs" in 1991 (see A.M. Best Company, 1992, p. 67).

Concern about financial strength has led insurers to hold financial assets in addition to those held as reserves for the expected value of liabilities. This *surplus* amounts to collateral held for policyholders. When losses are higher than expected, surplus is available for claim payments. U.S. life-health and property-casualty insurance companies held around \$500 billion of surplus in 1999 (A.M. Best). This section shows how insurers economize on this costly collateral by constricting quantity in high-risk market segments via price increases. This constriction occurs in both competitive and noncompetitive situations, as the main reason for avoiding risk relates to consumer preferences rather than market power.

Formally, consider a market with N consumer types. Consumer utility in the ith market segment is defined over the price p_i and quality $q: \; U_i(p_i, q)$. Product quality corresponds to the insurer's financial strength, or its ability to meet the obligations promised in the contract.[4] Demand in the ith market is denoted by $y_i(p_i, q)$ and is assumed to be decreasing in price and increasing in quality. Up-front production costs are represented by $c(y_1, \ldots, y_N)$. The total assets held by the insurance company (just after the production of contracts) amount to revenues minus up-front costs plus capital,

$$\sum p_i y_i - c(y_1, \ldots, y_N) + R, \tag{1}$$

where R is the initial capital contributed by firm owners. These assets are invested at the risk-free rate r_{f}.

At the end of one period, consumers submit claims. Total claims are represented by the random variable $\mathbf{L}$ and are distributed according to the amounts sold in each of the N markets, with

$$\frac{\mathrm{E}[\mathbf{L}]}{1 + r_{\mathrm{f}}} = \sum \mu_i y_i, \tag{2}$$

where μ_i represents the expected discounted average claim per unit of demand in the ith market. If the claims submitted exceed the firm's assets, the firm will pay only to the extent of its asset holdings. The "safety margin" contained in the balance sheet is called *surplus* and is defined as the excess of assets over discounted expected claims and the discounted value of any distributions to shareholders θ that will occur before the payment of claims:

$$S = R + \sum p_i y_i - c(y_1, \ldots, y_N) - \sum \mu_i y_i - \theta. \tag{3}$$

Shareholder distributions are assumed to be fixed in advance. The portion of claims not paid when the company defaults can thus be constructed as the random variable

$$\mathbf{D} = \max[0, \mathbf{L} - \mathrm{E}[\mathbf{L}] - S(1 + r_{\mathrm{f}})], \tag{4}$$

[4] A simplification here is that quality is perceived similarly by all consumers, as might be the case if quality were captured in company ratings such as those issued by A.M. Best. Technically, customers of the same insurer might be holding insurance of differing quality when state dependence is considered, despite the absence of any claim seniority. For example, a high-risk consumer's losses could be strongly correlated with insurer default. This complication is ignored in the following analysis.

with the expected value E[**D**] being a function of surplus S and the quantities $y_1, \ldots, y_N$.

The distribution of **D** determines the quality of the insurance contract. Quality can thus be expressed as $q(S, y_1, \ldots, y_N)$. Quality is obviously increasing in surplus, which makes default less likely and less severe whenever it occurs. The relationship between quantity in the ith market and quality depends on the quantities supplied in other markets and the expected covariation between the losses in the ith market and those in the other markets. This definition of quality is consistent with a variety of specifications of consumer preferences. For example, quality could be defined by aspects of the distribution of **D**, such as the probability of default and its expected severity. It could also be defined as a financial strength rating, such as an A.M. Best rating.

Capital costs and the possibility of default complicate profit maximization. If δ is the discounted cost of holding capital, expected discounted profits are

$$\sum_{i=1}^{N} p_i y_i - c(y_1, \ldots, y_N) - \sum \mu_i y_i + \frac{\mathrm{E}[\mathbf{D}]}{1 + r_\mathrm{f}} - \delta R. \tag{5}$$

The cost of holding capital[5] is the required rate of return on insurance equity minus the rate of return that the firm obtains on invested assets, adjusting for taxation and other frictional costs. Initially, let δ be fixed, implying that insurance risk is independent of returns on other assets. In general, δ can depend on the insurer's portfolio of policyholders. Section 3 discusses this issue.

To maximize expected profits, the firm solves

$$\max_{\{p_i\},R} \left\{ \sum_{i=1}^{N} (p_i - \mu_i) y_i(p_i, q) - c(y_1, \ldots, y_N) + \frac{\mathrm{E}[\mathbf{D}]}{1 + r_\mathrm{f}} - \delta R \right\}. \tag{6}$$

The first order condition for the choice of the ith price is

$$y_i + \left(p_i - \mu_i - \frac{\partial c}{\partial y_i}\right) \frac{\partial y_i}{\partial p_i} + \frac{\mathrm{d}\mathrm{E}[\mathbf{D}]/\mathrm{d}p_i}{(1 + r_\mathrm{f})} + \sum_{j=1}^{N} \left(p_j - \mu_j - \frac{\partial c}{\partial y_j}\right) \frac{\partial y_j}{\partial q} \frac{\mathrm{d}q}{\mathrm{d}p_i} = 0. \tag{7}$$

The first two terms represent the familiar pricing effects. The third term captures the effect of the price change on savings associated with default. The last term represents the marginal change in profit induced by any change in product quality. By changing the ith price, the firm affects product quality directly through its effect on profit and through its effect on demand of the ith type. The change also has a "ripple" effect on quality (see below): as the quantities in other market segments respond to the initial shift in the ith price, quality changes. The main lesson is that the firm cannot consider pricing decisions in a vacuum when consumers care about quality and capital is costly to hold. Serving one class of customers has an impact on demand by other customers.

[5] Note that surplus is unaffected by the cost of capital, since payments to shareholders (other than the fixed distributions θ) occur after payments to policyholders. In addition, δ is an *expected* cost in the sense that the actual payments to stakeholders will depend on results.

The ultimate response of quality to a change in price,

$$\frac{\mathrm{d}q}{\mathrm{d}p_i} = \frac{\dfrac{\partial q}{\partial S}\left(y_i + \left[p_i - \mu_i - \dfrac{\partial c}{\partial y_i}\right]\dfrac{\partial y_i}{\partial p_i}\right) + \dfrac{\partial q}{\partial y_i}\dfrac{\partial y_i}{\partial p_i}}{1 - \dfrac{\partial q}{\partial S}\sum\left[p_j - \mu_j - \dfrac{\partial c}{\partial y_j}\right]\dfrac{\partial y_j}{\partial q} - \sum\dfrac{\partial q}{\partial y_j}\dfrac{\partial y_j}{\partial q}}, \tag{8}$$

is complex. As is the case in other models in which quality depends on demand, the relation between price and quality may not be uniquely defined and may be positive or negative in different regions. The sign of Eq. (8) determines the impact of quality on the pricing decision in the ith market through the marginal condition, Eq. (7). For example, if quality is increasing in the ith price, the firm must balance gains associated with increasing quality with the loss of business resulting from the price increase in the ith market.

The optimality condition for the choice of capital is

$$-\delta + \frac{\frac{\mathrm{dE}[\mathbf{D}]}{\mathrm{d}R}}{(1+r_f)} + \sum_{j=1}^{N}\left(p_j - \mu_j - \frac{\partial c}{\partial y_j}\right)\frac{\partial y_j}{\partial q}\frac{\mathrm{d}q}{\mathrm{d}R} \leq 0. \tag{9}$$

The first term is the marginal cost of capital, while the next two terms make up the marginal impact of capital on contract revenues and expenses. The condition describes a balancing of the costs of placing risk with consumers (through exposure to default) with the costs of placing risk with investors by holding more capital. The chosen balance depends both on the value that consumers place on quality and on the cost of providing that quality. It is instructive to consider two polar cases.

First, suppose that consumers did not care about quality, as might be the case if guaranty fund protection were complete. In this case, $\partial y_i/\partial q = 0$ for all i. Firms would make no effort to avoid default and would hold no capital—the benefit to adding capital (the last term in Eq. (9)) disappears when consumers do not care about default. The last term in Eq. (7) also disappears, indicating that risk is *rewarded* in the sense that markets with higher marginal contributions to E[**D**] have lower prices. The intuition is simple. When consumers are indifferent to default, the firm will place all risk with consumers.

Next, suppose consumers do care about quality but that there is no cost associated with holding capital ($\delta = 0$). The firm then balances the second and third terms of Eq. (9). If the consumers are "collectively" risk-averse, the firm will hold enough capital to guarantee solvency. Without default, however, the third and fourth terms in Eq. (7) disappear, and the standard monopoly pricing rules apply. Prices are unaffected by risk, as default is no longer at issue. All risk is costlessly placed with investors.

In the general case, consumers care about quality, and there is a cost to placing risk with investors. The firm then acts as an intermediary, splitting the burden of risk-bearing and pricing policies to reflect marginal contributions to default risk. This paper assumes that the firm itself faces no costs of financial distress and that investors are well diversified, caring about insurance risk only to the extent that it relates to the returns on other assets. Relaxing this assumption will lead to additional pricing factors and motivations for risk penalties, such as those discussed in Froot and Stein (1998) and Doherty (1991).

2.1. Pricing in a portfolio model

A special case of the profit maximization problem arises when claims are distributed normally. This is a convenient example, as it allows both a closed-form expression for Eq. (7) and the study of competitive pricing as a limiting case of the profit-maximizing rule. The objective of this section is simply to show how insurance risk determines contract pricing and capital requirements, independently of its connection to the broader securities market.

Formally, suppose there is uncertainty in the *average* claim for each of the N classes of consumers, with the average claim in the ith class (denoted a_i) being distributed normally with mean μ_i and variance σ_{ii}. The covariance between the average claims in the ith and jth classes is denoted by σ_{ij}. There is also uncertainty at the individual level. A consumer in the ith class experiences a normally distributed claim with mean a_i (the class average) and variance ρ, where ρ is identical across classes. Claims are i.i.d. across consumers within a class. The variance of the insurer's total claim distribution is

$$\sigma = \sum_i \sum_j y_i y_j \sigma_{ij} + \sum_i y_i \rho. \tag{10}$$

The first term represents the contribution of aggregate uncertainty to total portfolio variance, while the second represents the contribution of *process* risk to total portfolio variance. Aggregate uncertainty refers to uncertainty in the *expected* average costs for the insured population, and process risk refers to the risk that the company's portfolio averages will differ from the population averages due to chance.

Information on S and σ is sufficient to produce the relevant moments of the normal distribution and the truncated normal distribution, so both expected default costs and quality may be rewritten as a function of these two variables. For quality, assume $q(S, y_1, \ldots, y_N) \equiv \tilde{q}(S, \sigma)$, with $\tilde{q}_\sigma < 0$.

Finally, assume that the insurer's production cost function is an increasing function of total demand:

$$c(y_1, \ldots, y_N) = c\left(\sum_{i=1}^{N} y_i\right). \tag{11}$$

The appendix shows that with the assumptions above, Eq. (7) implies

$$\frac{p_i\left(1 + \dfrac{1}{\varepsilon_i}\right) - \mu_i - c'}{\sum_j s_j \left[p_j\left(1 + \dfrac{1}{\varepsilon_j}\right) - \mu_j\right] - c'} = \frac{\mathrm{cov}_{i,z} + \dfrac{\rho}{2\sum_j y_j}}{\mathrm{var}_z + \dfrac{\rho}{2\sum_j y_j}}, \tag{12}$$

where ε_i is the price elasticity of demand in the ith market, $s_i = y_i / \sum_j y_j$ is the share of the ith consumer type in the insurer's portfolio, $\mathrm{cov}_{i,z}$ is the covariance of the aggregate risk component in the ith class with the aggregate risk component of the insurer's portfolio, and var_z is the variance associated with aggregate risk in the insurer's portfolio.

If the insurer is sufficiently large, we can ignore process risk and set $\rho/2\sum_j y_j \approx 0$. In this case, Eq. (12) implies

$$p_i\left(1+\frac{1}{\varepsilon_i}\right)-\mu_i-c'=\beta_i\left(\sum_j s_j\left[p_j\left(1+\frac{1}{\varepsilon_j}\right)-\mu_j\right]-c'\right), \tag{13}$$

where β_i is defined as $\text{cov}_{i,z}/var_z$. With competitive markets, this relation approaches

$$p_i-\mu_i-c'=\beta_i(\bar{p}-\bar{\mu}-c'), \tag{14}$$

in the limit as $\varepsilon_i \to \infty$ for all i, where $\bar{p}=\sum_j s_j p_j$ and $\bar{\mu}=\sum_j s_j\mu_j$. The "markup" over marginal costs in the ith market is the product of a "beta" and the mean markup of the insurer. Since $\mu_i + c'$ and $\bar{\mu}+c'$ are effectively the riskless rates of return in the model, this relation is similar in *appearance* to the familiar CAPM pricing equation.

Yet a crucial distinction between Eq. (14) and the CAPM is that the pricing is driven by covariation across insurance markets, rather than covariation with the overall securities market. In this example, the uncertainty in the insurance markets is independent of capital market returns, but prices *still* feature risk penalties because of the consumer concern for solvency. The institutional details of the insurance market—production in limited liability companies with costly capital—cause the logic of equilibrium asset pricing models to fail when applied to insurance.

Viewed from the perspective of the firm, the beta in Eq. (13) is a firm-specific beta. Because product quality is determined by the solvency of the company itself, the relevant beta for pricing reflects the insurer's portfolio of risk. If firms hold different consumer portfolios (as appears to be the case in practice—many firms specialize by region or line), both the "fair" price and the chosen markup for a consumer would vary across firms. Of course, in theory, competitive pressures could lead prices and portfolios to equalize across insurers, in which case the betas would be identical across firms.

In principle, the sign of $\sum_j s_j p_j(1+1/\varepsilon_j)-\bar{\mu}-c'$ could be positive or negative. More risk means higher profits when consumers do not care about solvency ($U_q = 0$), since the fraction of claim payments avoided through default is increasing in risk. In this case, $\sum_j s_j p_j(1+1/\varepsilon_j)-\bar{\mu}-c'<0$, and risk would be *rewarded* rather than penalized. The firm would hold no capital and place all default risk with consumers. High-risk market segments would have *lower* markups than average. When consumers do care about solvency ($U_q>0$), the gains from default savings are offset by the harmful effects of risk on demand. If the latter effect is stronger than the former, $\sum_j s_j p_j(1+1/\varepsilon_j)-\bar{\mu}-c'>0$, and risk is penalized.

The importance of insolvency risk in the portfolio problem determines how far optimal pricing will deviate from the standard rules for monopoly and competition. For example, suppose that there were no capital holding costs and that the firm held enough capital to rule out default. Prices then would be set according to the standard monopoly pricing rule, $p_i(1+1/\varepsilon_i)-\mu_i-c'=0$, in all markets. Thus, $\sum_j s_j p_j(1+1/\varepsilon_j) - \bar{\mu} - c' = 0$, and prices would not vary according to risk. On the other hand, when $\sum_j s_j p_j(1+1/\varepsilon_j) - \bar{\mu} - c' > 0$, prices will vary according to risk by Eq. (13). The size of this average deviation corresponds to the importance of default risk considerations in the firm's problem and is determined by consumer attitudes and the cost of capital. If the average deviation is small, prices will not differ substantially from the standard rules, and the penalty for risk is small. If it is large, the penalty for risk will also be large.

The same logic applies to competitive pricing in Eq. (14). When default risk is nonexistent, price will be set according to the standard $p_i = \mu_i + c'$ rule in every market. With default risk, however, prices deviate from the standard rule according to risk, with the size of the average deviation determining the extent to which risk is penalized or rewarded. In this sense, high-risk markets experience the largest changes when default considerations, through changes in the cost of capital or consumer attitudes toward solvency, become important.

How can the competitive pricing in this example be decomposed into standard costs of production? To see this, we study the allocation of capital by line of insurance.

2.2. Competitive capital allocation

Fair pricing of insurance, based on Eq. (5), implies

$$\bar{p} = \frac{\sum p_i y_i}{\sum y_i} = \frac{c(y_1, \ldots, y_N) + \sum \mu_i y_i - \frac{\mathrm{E}[\mathbf{D}]}{1+r_\mathrm{f}} + \delta R}{\sum y_i}. \tag{15}$$

The numerator on the right shows the expense components of the markup. The first is the contribution of production costs, the second is expected loss, the third is an adjustment for claims that are not paid when the company defaults, and the fourth is the capital cost. It is easy to allocate the first two expense types to individual markets, but the allocation of the last two—capital costs and default savings—is not obvious. A competitive price for insurance in the ith market clearly must contain a capital cost component, but what is it?

We start by observing that the definition of quality implies a marginal surplus allocation rule based on the implicit function $\hat{S}(y_1, \ldots, y_N; q)$, the amount of surplus necessary to deliver a fixed level of quality q. Associated with this function is an implicit capital function $\hat{R}(p_1, \ldots, p_N, y_1, \ldots, y_N; q)$ satisfying

$$\hat{S}(y_1, \ldots, y_N; q) = \sum (p_i - \mu_i) y_i - c(y_1, \ldots, y_N) - \theta + \hat{R}(\theta, p_1, \ldots, p_N, y_1, \ldots, y_N; q). \tag{16}$$

The function $\partial \hat{S}/\partial y_i$ reveals how much surplus is necessary to "support" a marginal expansion of coverage in the ith market segment, while $\partial \hat{R}/\partial y_i$ translates this into a marginal increase (or decrease) in capital, given a set of prices.

Going further, marginal cost pricing implies

$$p_i = \frac{\partial c}{\partial y_i} + \mu_i - \frac{\mathrm{dE}[\mathbf{D}]/\mathrm{d}y_i}{1+r_\mathrm{f}} + \delta \frac{\partial \hat{R}}{\partial y_i}, \tag{17}$$

where $\mathrm{dE}[\mathbf{D}]/\mathrm{d}y_i = (\partial \mathrm{E}[\mathbf{D}]/\partial S)\,(\partial \hat{S}/\partial y_i) + \partial \mathrm{E}[\mathbf{D}]/\partial y_i$. Using Eq. (16) and (17), marginal cost—the right-hand side of Eq. (17)—can be shown to be

$$\frac{\partial c}{\partial y_i} + \mu_i - \frac{\dfrac{\mathrm{dE}[\mathbf{D}]}{\mathrm{d}y_i}}{1 + r_\mathrm{f}} + \delta \left[\frac{\dfrac{\partial \hat{S}}{\partial y_i} + (\mathrm{dE}[\mathbf{D}]/\mathrm{d}y_i)/(1 + r_\mathrm{f})}{1 + \delta} \right]. \tag{18}$$

The bracketed term is $\partial \hat{R}/\partial y_i$, the marginal increase of capital required to keep quality constant. The magnitude of the requirement depends on the nature of risk in the ith market and how it relates to the risk borne in other markets. Thus, the true marginal cost of coverage in the ith market can be broken into marginal production cost, marginal claims costs (net of marginal changes in default savings), and marginal cost associated with a capital requirement.

Applying this to the portfolio example under competition, consider the case in which quality is a one-to-one function of the probability of default. Assume zero shareholder distributions ($\theta = 0$), linear production costs, and zero profits. This implies that $S = (1 + \delta)R - \mathrm{E}[\mathbf{D}]$, and the term-by-term analog of Eq. (18) can be shown to be

$$c' + \mu_i - \frac{\beta_i \mathrm{E}[\mathbf{D}]/(1 + r_\mathrm{f})}{\sum y_i} + \delta \frac{\beta_i R}{\sum y_i}. \tag{19}$$

In this example, the capital required at the margin to hold quality constant varies directly in proportion to β_i. Since the weighted average of the betas will equal one, capital is allocated to each market on a per unit basis according to β_i—a simple "beta" rule. Since high-beta markets are capital-intensive, they experience greater increases in costs when the cost of capital rises.

This is but one possible capital allocation rule. Myers and Read (2001) offer a capital allocation rule based on a default insurance concept. A comparable rule would be obtained in this example by assuming that quality is determined by the expected value of defaulted claims divided by the expected value of liabilities, rather than the probability of default. The important point is that, in general, the appropriate capital allocation rule is driven by consumer attitudes toward risk. In principle, the rule could be affected by any aspect of the distribution of defaulted claims, in addition to those already mentioned.

In summary, Eqs. (14) and (19) show that even a competitive insurer will have "markups" over expected losses that vary across market segments. High prices in high-risk segments result from high marginal capital requirements, which are driven by consumer demand for quality. These hidden capital requirements may push price to a level in excess of what seems "fair" or "profit-maximizing" in a given market segment, but are indeed necessary to mitigate the externalities generated by that segment on the overall insurance pool and must be taken into account when evaluating the price.

3. Asset pricing theory and costly capital

Much of the existing literature develops prices based on the perspective of the capital market (see, e.g., Fairley, 1979). Insurance is priced to offer shareholders a fair expected return in comparison with other investment opportunities, as determined by the expected relationship between returns on insurance policies and overall capital market returns. Firm-specific risk does not matter, as investors are able to diversify. However, as first argued by Doherty and Tinic (1981), this approach ignores the perspective of the policyholder. Section 2 demonstrates how policyholder concern about solvency engenders pricing that appears to contradict the usual lessons from standard asset pricing models. This section derives the cost of capital and connects the consumer-based approach to risk pricing with the investor-based approach.

The connection hinges on the nature of the cost of capital. If the cost depends solely on the relation between insurance market risk and capital market risk, contract pricing will reflect risk penalties based solely on that relation. On the other hand, if the cost is driven by other factors, insurers may be under pressure to economize on capital, and this will lead to risk penalties in addition to those based on the relationship between insurance risk and capital market risk. To see this, consider the following simple model of the cost of insurance capital under competition.[6]

Interpret R as the equity contributed by stakeholders. Recall that the insurer collects premiums and pays production expenses at the start of the period. Hence, the insurer invests $\sum_i p_i y_i - c(y_1, \ldots, y_N) + R$ at the start of the period and pays $\mathbf{L}$ at the end of the period, minus any savings that are realized in the event of default. The return on the insurance equity investment is

$$\left(\frac{\left[\sum_i p_i y_i - c(y_1, \ldots, y_N)\right](1 + r_{\mathrm{f}}) - \mathbf{L} + \mathbf{D}}{R} + r_{\mathrm{f}}\right)(1 - \tau), \tag{20}$$

where τ is the corporate income tax rate. Investors must be compensated for risk, as well as for any frictional costs borne. These latter costs are denoted by f and reflect additional monitoring, agency, or liquidity costs associated with the insurance company investment. The required rate of return is then expressed as

$$\mathrm{E}[\mathbf{r}_{\mathrm{C}}] = f + r_{\mathrm{f}} + (b_D - b_L)(1 - \tau)(\mathrm{E}[\mathbf{r}_{\mathrm{m}}] - r_{\mathrm{f}}), \tag{21}$$

where $\mathrm{E}[\mathbf{r}_{\mathrm{m}}]$ is the expected rate of return on the equity market, b_D is the CAPM beta associated with $\mathbf{D}/R$, and b_L is the CAPM beta associated with $\mathbf{L}/R$. Note that

$$b_D = \frac{(1/R)\,\mathrm{cov}(\mathbf{D}, \mathbf{r}_{\mathrm{m}})}{\mathrm{var}(\mathbf{r}_{\mathrm{m}})}, \; b_L = \frac{(1/R)\mathrm{cov}(\mathbf{L}, \mathbf{r}_{\mathrm{m}})}{\mathrm{var}(\mathbf{r}_{\mathrm{m}})}. \tag{22}$$

[6] The development that follows is a greatly simplified model in many respects, intended only to keep the paper self-contained and illustrate the basic contribution of taxes and other frictional costs. Details are ignored for the sake of simplicity and transparency. For derivations with more sophistication with respect to tax and cash flow issues, see D'Arcy and Doherty (1988) and Cummins and Harrington (1987) [especially Myers and Cohn (1987) and Hill and Modigliani (1987)], as well as Derrig (1994).

The cost of capital is the difference between the rate of return required by the capital market, $\mathrm{E}[\mathbf{r}_{\mathrm{C}}]$, and the return that can be earned on invested assets, $r_{\mathrm{f}}(1-\tau)$, discounted and adjusted for taxation. For each unit of capital held,

$$\frac{\mathrm{E}[\mathbf{r}_C]-r_{\mathrm{f}}(1-\tau)}{(1+r_{\mathrm{f}})(1-\tau)}=\frac{f+\tau r_{\mathrm{f}}+(b_D-b_L)(1-\tau)\big(\mathrm{E}[\mathbf{r}_{\mathrm{m}}]-r_f\big)}{(1+r_{\mathrm{f}})(1-\tau)} \tag{23}$$

must be recovered from consumers before taxes to make up for this difference. Thus, using Eqs. (22) and (23), total capital costs are

$$\delta R=\frac{(f+\tau r_{\mathrm{f}})R}{(1+r_{\mathrm{f}})(1-\tau)}+\left[\frac{\mathrm{cov}(\mathbf{D},\mathbf{r}_{\mathrm{m}})-\mathrm{cov}(\mathbf{L},\mathbf{r}_{\mathrm{m}})}{\mathrm{var}(\mathbf{r}_{\mathrm{m}})}\right]\frac{(\mathrm{E}[\mathbf{r}_{\mathrm{m}}]-r_{\mathrm{f}})}{(1+r_{\mathrm{f}})}. \tag{24}$$

This represents a decomposition of the cost of capital into a frictional component and a risk component. The second term on the right-hand side—the risk component—arises from the relation between insurance liabilities and capital market returns. It represents the compensation investors demand for bearing insurance risk according to its relation with the capital market and depends on the amount of capital held only to the extent that leverage affects the split of losses between **L** and **D**. The first term on the right-hand side—the frictional component—relates to tax and frictional costs. It is independent of the nature of insurance risk and is directly proportional to the amount of capital held. With taxes, capital is costly for firms to hold even when aggregate losses are independent of the capital market return and frictional costs are zero. Every dollar of capital held earns r_{f}, and these earnings are taxed at a rate of τ. In this case, investors demand a return of r_f on equity, but the firm can only obtain a return of $(1-\tau)r_f$ through investment.

When there are frictional or tax costs associated with holding capital, firms will manage risk to economize on capital. This leads to risk penalties in addition to those imposed by investors for bearing risk. In other words, risk that threatens the solvency valued by consumers will be penalized in the firm's prices, whether or not the capital market penalizes that risk. Indeed, these "consumer" penalties may be more important than the capital market penalties when it comes to insurance pricing. Froot et al. (1995) and others have argued that catastrophe losses are small relative to financial wealth and uncorrelated with returns on other assets. More broadly, empirical studies of industry underwriting betas (see, e.g., Cummins and Harrington, 1985) have yielded estimates close to zero. This suggests that the cost of insurance capital is not driven by covariation with asset market returns, leaving taxation and other frictions as the main cost drivers.

3.1. Pricing with an endogenous cost of capital

This section considers how pricing changes when the insurer's mix of business affects the cost of capital. It extends the model of Section 2 by incorporating investor attitudes toward risk.

Formally, consider the situation when the cost of capital depends on capital, surplus, and quantities: $\delta(R, S, y_1, \ldots, y_N)$. The optimization of Eq. (6) now yields the following first order condition for choice of the ith price:

$$y_i + x_i \frac{\partial y_i}{\partial p_i} + \sum_{j=1}^{N} x_j \frac{\partial y_j}{\partial q} \frac{\mathrm{d}q}{\mathrm{d}p_i} - \frac{\partial \delta}{\partial S} \frac{\mathrm{d}S}{\mathrm{d}p_i} R + \frac{\mathrm{dE}[\mathbf{D}]/\mathrm{d}p_i}{(1 + r_\mathrm{f})} = 0, \tag{25}$$

where $x_i = p_i - \mu_i - \partial c/\partial y_i - (\partial\delta/\partial y_i)R$. This is identical to the original condition, Eq. (7), with additional contributions to marginal cost from the effects of the price change on the cost of capital.

This suggests an additional reason why markups may be high in insurance market segments with significant uncertainty in the average loss. If the uncertainty is negatively correlated with capital market returns, the insurer gains by raising price in this segment—exposure to the "high beta" uncertainty is reduced, and the insurer's cost of capital falls. Hence, the insurer is rewarded for raising price in such a market twice—once by consumers, who appreciate the effects on quality, and once by suppliers of capital, who demand a lower return due to the reduced exposure to systematic risk.

To see the significance of Eq. (25) more clearly, consider the normal risk example of Section 2.1. This example can be extended to include an endogenous cost of capital with some modifications. Specifically, assume that θ, the amount distributed to shareholders before claims are paid, equals total capital costs δR. Then,

$$S = (1 - \delta)R + \sum_{i=1}^{N} (p_i - \mu_i) y_i - c\left(\sum_{i=1}^{N} y_i\right). \tag{26}$$

The appendix derives the following endogenous capital cost analogs of Eqs. (13) and (14) in this setting:

$$p_i\left(1 + \frac{1}{\varepsilon_i}\right) - \mu_i - c' - \frac{\partial\delta}{\partial y_i} R = \beta_i \left(\sum_j s_j \left[p_j\left(1 + \frac{1}{\varepsilon_j}\right) - \mu_j - c' - \frac{\partial\delta}{\partial y_j} R\right]\right), \tag{27}$$

$$p_i - \mu_i - c' - \frac{\partial\delta}{\partial y_i} R = \beta_i \left(\bar{p} - \bar{\mu} - c' - \sum s_j \frac{\partial\delta}{\partial y_j} R\right). \tag{28}$$

Thus, the competitive markup over marginal loss and production costs in any market segment can be decomposed into a component reflecting market risk and a component reflecting firm-specific risk, as in

$$p_i - \mu_i - c' = \frac{\partial\delta}{\partial y_i} R + \beta_i \left(\bar{p} - \bar{\mu} - c' - \sum s_j \frac{\partial\delta}{\partial y_j} R\right). \tag{29}$$

The first term on the right-hand side reflects how changes in demand in the ith market affect the cost of holding capital. In other words, it represents the marginal change in the return required by shareholders. The second term reflects the impact of demand changes in the ith market on product quality.

As before, the magnitude of the average deviation $\left(\bar{p} - \bar{\mu} - c' - \sum s_j(\partial\delta/\partial y_j)R\right)$ corresponds to the importance of default risk in the consumer market and is a measure of the effective risk aversion of the firm.[7] When risk does not matter to consumers and default does not affect profits, the second term disappears. In this case, the competitive markup equals the marginal change in the cost of capital, $(\partial\delta/\partial y_i)R$, as only stakeholders are affected by changes in the composition of risk. Pricing is then consistent with the predictions of traditional capital market models. When risk does matter to consumers, the average deviation from actuarial pricing will be positive $(\bar{p} - \bar{\mu} - c' - \sum s_j(\partial\delta/\partial y_j)R > 0)$, and markups reflect marginal effects on shareholders *and* consumers.

4. Capital costs in practice

Section 2 shows how, in theory, differences in capital allocations generate price differences across insurance markets. This section presents evidence on the empirical significance of capital in insurance markets. It shows that capital holdings vary across lines of insurance, and that this variation is associated with corresponding variation in prices. It goes on to estimate the likely effect of capital costs on catastrophe contract pricing.

As a matter of economics, the capital held by insurers will be determined by a balancing of the value consumers place on capital with the cost of holding that capital. Table 1 shows the 1998 premium-weighted ratings distribution for the industry and for several lines of business within the industry (companies are defined by A.M. Best to "predominate" in a line of business based on premium distribution). Market forces have evidently led to strong average capitalization, well in excess of regulatory requirements. More than 90% of premiums were written in insurers rated either Superior (A++, A+) or Excellent (A, A−) by A.M. Best. There appears to be an especially strong demand for quality in the reinsurance business, which is consistent with the absence of guaranty fund protection in this line.

Of course, it could be argued that capital is being held for reasons relating to other costs of financial distress and that the high ratings are a by-product of such decisions, rather than a response to consumer demands. It is difficult to distinguish these interpretations empirically, but Doherty and Phillips (2002) find that insurers added capital in response to increasing rating stringency in the 1990s. This suggests that the rating is in fact the *target* of capital management.

How much do consumers pay for capital and the associated insurance quality? As a matter of arithmetic, the *capital cost ratio* is defined by a decomposition of the fair premium from Eq. (15):

$$\frac{\delta R}{\sum p_i y_i} = \frac{\sum p_i y_i - c(y_1, \ldots, y_N) - \sum \mu_i y_i + \mathrm{E}[\mathbf{D}]/(1 + r_\mathrm{f})}{\sum p_i y_i}. \tag{30}$$

[7] This decomposition of pricing into systematic and firm-specific components bears similarity to the results of Froot and Stein (1998). In their model, costs associated with external financing cause hurdle rates to reflect both systematic and firm-specific components, with the importance of the latter being determined by the degree of firm risk aversion.

Table 1
Premium distribution by 1998 best rating.

Line	1998 A.M. Best rating premium distribution (%)				
	A++, A+	A, A−	B++, B+	Vulnerable	Unrated
Consolidated industry	56.4	36.7	3.4	0.5	3.0
Accident and health	0.0	82.1	10.0	0.0	7.9
PP Auto and homeowners	61.9	34.2	3.3	0.2	0.4
Commercial Auto	46.5	42.3	3.0	3.3	5.0
Commercial casualty	55.3	42.3	2.0	0.2	0.2
Fidelity and surety	38.6	59.0	0.0	1.2	1.3
Property	14.6	78.0	6.2	0.3	0.9
Credit	20.5	69.9	4.2	1.3	4.1
Medical malpractice	0.0	88.3	4.7	1.9	5.2
Reinsurance	74.4	25.0	0.5	0.0	0.1
Financial guaranty	0.0	0.0	0.0	0.0	100.0

This is an accounting profit margin, or the portion of the premium that remains after losses and production expenses are deducted. It represents the residual part of the premium used to pay for capital.

Table 2 presents capital cost ratio estimates for the industry and lines, based on a ten-year (1989–1998) averaging of A.M. Best data. To be in the tables, the line had to be included by A.M. Best in the "Balance Sheet and Summary of Operations" section for each of the years in question, which served as the basis for the calculations. Surplus and net income were adjusted for discounting, with the adjustment based on a discounting of both year-start and year-end loss reserves using the 1999 Schedule P for estimates of the timing of loss payments and the 1989–1999 average return on invested assets (including realized and unrealized gains) as the discount rate.[8] The *capital cost ratio* is estimated as (adjusted net income plus unrealized gains plus taxes minus investment income attributable to surplus) divided by (total revenues minus policyholder dividends minus investment income attributable to surplus). Investment income attributable to surplus is calculated as the product of the return on invested assets and adjusted surplus. The *adjusted capital-to-premium ratio* is the ten-year average of the ratio of adjusted surplus to net premiums written. Additional details about the calculations are available from the author.

Over this period, the industry featured a modest capital-to-premium ratio (close to one) and a modest return on capital, which meant that residual costs were only a small portion of the premium. There was considerable variation in both profitability and capital leverage across lines of insurance. Some lines featured double-digit capital cost ratios, driven by profitability in some cases (e.g., Credit), by capital leverage in some cases (e.g., Reinsurance), and by both in others (e.g., Financial

[8] Using the treasury yield curve as the basis for discounting did not have a significant impact on the results.

Table 2
Adjusted capital-to-premium ratio and capital cost ratio averages for 1989–1998.

Line	Adjusted capital-to-premium ratio	Capital cost ratio (%)
Consolidated industry	1.1	4
Accident and health	0.6	0
PP Auto and homeowners	0.8	3
Commercial Auto	0.8	5
Commercial casualty	1.1	1
Fidelity and surety	1.1	11
Property	1.2	3
Credit	1.4	21
Medical malpractice	2.1	17
Reinsurance	2.6	13
Financial guaranty	4.0	63

Guaranty). It is evident from the table that residual costs were rising in capitalization—payments for capital can be a substantial part of the price when capitalization is high.

To assess the effect of capital costs on catastrophe insurance pricing, it is necessary to determine how much capital is being held for catastrophe contracts. One indicator is the capitalization of companies that specialize in providing catastrophe insurance. Recent figures for Bermuda's catastrophe and excess liability reinsurance companies show average statutory capitalization in the neighborhood of three to five times premiums (Bermuda Insurance Update #36, Winter, 2001). Some underwriters hold even more. For example, in 1999, National Indemnity held statutory capital at about 40 times premiums (A.M. Best). We can obtain another indicator by applying a risk-based capital methodology to earthquake insurance, a line isolated from other property business in industry statistics. Although earthquake insurance is not a line specifically addressed by the NAIC, the spirit of the model is to require capitalization in each line of insurance that would guarantee solvency for the worst year in recent history. The relevant year for earthquake insurance is 1994, when the combined ratio of the line's primary writers was 889% (A.M. Best). This implies a capital requirement of about eight times premiums.[9] Fig. 1 shows how the price impact associated with capital costs ($1/(1-\delta R/\sum p_i y_i)$) varies with the capital-to-premium ratio for $\delta = 0.05$, the approximate value for the reinsurance industry from Table 2. As shown, even a multiple of five implies a price impact of about 30%.[10]

[9] Of course, the NAIC methodology incorporates allowances for diversification (a discount of up to 30%), investment income, and company-specific development. For more information, see Laurenzano (1995).

[10] See Harrington and Niehaus (2000) for alternative estimates of the impact of capital costs (in particular, taxes) on catastrophe insurance pricing.

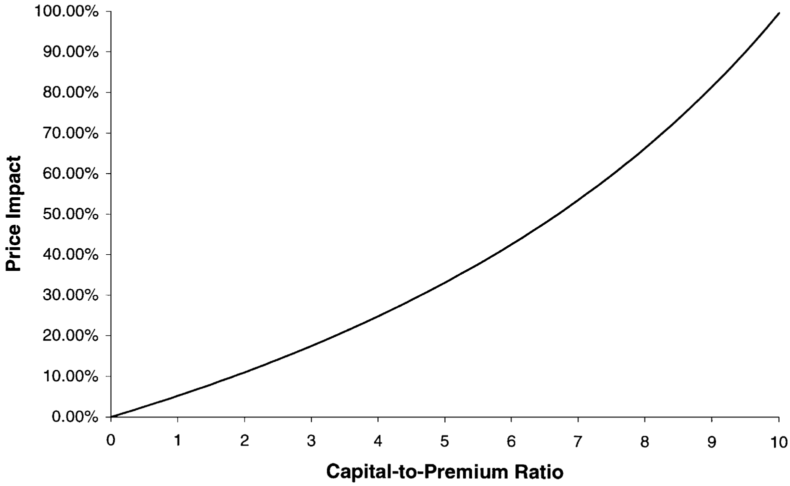

Fig. 1. Theoretical impact of capital costs on price with $\delta = 0.05$.

5. Conclusion

When solvency matters to consumers and capital is costly to hold, limited liability firms must manage risk and diversify. The paper developed market-based prices and marginal capital requirements under these conditions. The results imply that customers who bring severe externalities in the form of insurance risk will be penalized in a competitive market, with the penalties reflecting higher marginal capital requirements. However, because of the social aspect of the insurance product (meaning that the quality of the service provided to one customer depends on the prices charged and quantities delivered to other customers), high prices for "high-risk" consumers should not necessarily be interpreted as a market failure.

Catastrophe insurance is not the only market where solvency concerns and capital costs are important. The paper's findings are relevant for any market with significant uncertainty in average losses. For example, these results are important for understanding the short-term nature of health insurance contracts, where uncertainty in future medical costs may lead to high capital requirements for underwriting long-term contracts. The analysis may also be applied to patterns of trade across consumer groups within markets. Groups with expected losses that are especially sensitive to changes in economic conditions, technology, or the weather will have higher capital requirements and prices.

The paper demonstrates that capital costs can have a substantial impact on prices in some lines of insurance. When surplus holdings are large relative to premiums, even a modest per unit capital cost can lead to a significant price impact. In the case of catastrophe risk, financing approaches must find ways to mitigate the capital costs that appear to have stifled the market. Recent years have witnessed the introduction of alternative risk-financing methods—such as Bermuda catastrophe reinsurance and catastrophe bonds—that reduce the tax costs associated with the supporting capital. While it is too early to judge the ultimate impact of these alternatives, their failure to dominate the market suggests that there may be other important capital holding costs at work. Of course, the theoretical portion of the paper may be applied to any situation with capital costs, whether the driver is a tax cost, an agency cost, or another type of frictional cost. Understanding the exact nature of capital costs and calibrating their influence on market behavior are important areas for future research.

Appendix

A.1. Show that Eq. (7) implies Eq. (12) under the assumptions of Section 2.1

Recall that under normal risk we can express expected default costs and quality as a function of surplus and variance, as in $q(S,\sigma)$ (where I have dropped the accent for ease of presentation), and

$$\mathrm{E}[\mathbf{D}] = T(S,\sigma) = \left[1-\Phi\left(\frac{S}{\sqrt{\sigma}}\right)\right]\left[\sqrt{\sigma}\lambda\left(\frac{S}{\sqrt{\sigma}}\right)-S\right], \tag{A.1}$$

where $\lambda(x)=\phi(x)/(1-\Phi(x))$, $\phi(.)$ is the pdf of the standard normal distribution and $\Phi(.)$ is the standard normal cdf. Note that $-1<T_S<0$ and $T_\sigma>0$. Under these assumptions, Eq. (7) is rewritten as

$$\left[y_i+(p_i-\mu_i-c^{'})\frac{\partial y_i}{\partial p_i}+\sum_{j=1}^{N}(p_j-\mu_j-c')\frac{\partial y_j}{\partial q}\frac{dq}{dp_i}\right](1+T_S)+\ldots \tag{A.2}$$

$$\cdots T_\sigma\left[\sum_{j=1}^{N}2y_j\sigma_{ij}+\rho\right]\frac{\partial y_i}{\partial p_i}+T_\sigma\left[2\sum\sum y_j\frac{\partial y_k}{\partial q}\sigma_{kj}+\sum\frac{\partial y_j}{\partial q}\rho\right]\frac{\mathrm{d}q}{\mathrm{d}p_i}=0. \tag{A.3}$$

In addition,

$$\frac{\mathrm{d}q}{\mathrm{d}p_i}=\frac{q_\sigma\left[\sum_{j=1}^{N}2y_j\sigma_{ij}+\rho\right]\frac{\partial y_i}{\partial p_i}+q_S\left[y_i+(p_i-\mu_i-c')\frac{\partial y_i}{\partial p_i}\right]}{1-q_\sigma\left[2\sum y_j\frac{\partial y_k}{\partial q}\sigma_{kj}+\sum\frac{\partial y_j}{\partial q}\rho\right]-q_S\left[\sum(p_j-\mu_j-c')\frac{\partial y_j}{\partial q}\right]}. \tag{A.4}$$

Substituting and simplifying yield

$$H_1\left[y_i+(p_i-\mu_i-c')\frac{\partial y_i}{\partial p_i}\right]+H_2\left[\sum_{j=1}^{N}2y_j\sigma_{ij}+\rho\right]\frac{\partial y_i}{\partial p_i}=0, \tag{A.5}$$

where

$$H_1 = (1 + T_S) + \left(\frac{(1+T_S)\sum(p_j - \mu_j - c')\frac{\partial y_j}{\partial q} + T_\sigma\left[2\sum\sum y_j \frac{\partial y_k}{\partial q}\sigma_{kj} + \sum\frac{\partial y_j}{\partial q}\rho\right]}{1 - q_\sigma\left[2\sum\sum y_j\frac{\partial y_k}{\partial q}\sigma_{kj} + \sum\frac{\partial y_j}{\partial q}\rho\right] - q_S\left[\sum(p_j - \mu_j - c')\frac{\partial y_j}{\partial q}\right]} \right) q_S, \tag{A.6}$$

$$H_2 = T_\sigma + \left(\frac{(1+T_S)\sum(p_j - \mu_j - c')\frac{\partial y_j}{\partial q} + T_\sigma\left[2\sum\sum y_j \frac{\partial y_k}{\partial q}\sigma_{kj} + \sum\frac{\partial y_j}{\partial q}\rho\right]}{1 - q_\sigma\left[2\sum\sum y_j\frac{\partial y_k}{\partial q}\sigma_{kj} + \sum\frac{\partial y_j}{\partial q}\rho\right] - q_S\left[\sum(p_j - \mu_j - c')\frac{\partial y_j}{\partial q}\right]} \right) q_\sigma. \tag{A.7}$$

Working with Eq. (A.5) yields

$$H_1\left[p_i\left(1 + \frac{1}{\varepsilon_i}\right) - \mu_i - c'\right] + 2H_2\left[\sum_{j=1}^{N} y_j\sigma_{ij} + \frac{\rho}{2}\right] = 0. \tag{A.8}$$

Note that H_1 and H_2 do not depend on i. Multiplying by y_i and summing across i yields

$$H_1\left[\sum y_i\left(p_i\left(1 + \frac{1}{\varepsilon_i}\right) - \mu_i - c'\right)\right] + 2H_2\left[\sum\sum y_j y_i\sigma_{ij} + \sum y_i\frac{\rho}{2}\right] = 0. \tag{A.9}$$

First, notice that $H_1 \neq 0$. To see this, suppose the opposite. Since $0 > T_S > -1$, $T_\sigma > 0$, $q_S > 0$, and $q_\sigma < 0$, $H_1 = 0 \Rightarrow H_2 > 0$. But this implies a contradiction by Eq. (A.9). We thus need to consider only two cases.

Case 1: $H_1 \neq 0, H_2 \neq 0$.

Note that this implies that $\sum\left(y_i p_i(1 + 1/\varepsilon_i) - \mu_i - c'\right) \neq 0$. We use Eq. (A.8) and (A.9) to obtain

$$\frac{p_i(1 + \frac{1}{\varepsilon_i}) - \mu_i - c'}{\sum y_i\left(p_i\left(1 + \frac{1}{\varepsilon_i}\right) - \mu_i - c'\right)} = \frac{\sum_{j=1}^{N} y_j\sigma_{ij} + \frac{\rho}{2}}{\sum\sum y_j y_i\sigma_{ij} + \sum y_i\frac{\rho}{2}}. \tag{A.10}$$

Multiplying through by $\sum y_i$ and simplifying yield the desired result.

Case 2: $H_1 \neq 0, H_2 = 0$.

In this case, Eq. (A.8) implies $p_i(1 + 1/\varepsilon_i) - \mu_i - c' = 0$ for all i. Thus, the result holds trivially.

A.2. Show that Eq. (25) implies Eqs. (27) and (28) under the assumptions of Section 2.1 and 3.1

To simplify notation, set $\delta R = Z(R, S, y_1, \ldots, y_N)$. The first order condition is

$$y_i + \left(p_i - \mu_i - c' - \frac{\partial Z}{\partial y_i}\right)\frac{\partial y_i}{\partial p_i} + \sum_{j=1}^{N}\left(p_j - \mu_j - c' - \frac{\partial Z}{\partial y_j}\right)\frac{\partial y_j}{\partial q}\frac{\mathrm{d}q}{\mathrm{d}p_i} + (T_S - Z_S)\frac{\mathrm{d}S}{\mathrm{d}p_i} + \ldots \tag{A.11}$$

$$\cdots T_\sigma\left[\sum_{j=1}^{N} 2y_j\sigma_{ij} + \rho\right]\frac{\partial y_i}{\partial p_i} + T_\sigma\left[2\sum\sum y_j\frac{\partial y_k}{\partial q}\sigma_{kj} + \sum\frac{\partial y_j}{\partial q}\rho\right]\frac{\mathrm{d}q}{\mathrm{d}p_i} = 0. \tag{A.12}$$

Solve for $\mathrm{d}S/\mathrm{d}p_i$ and $\mathrm{d}q/\mathrm{d}p_i$:

$$(1 + Z_S)\frac{\mathrm{d}S}{\mathrm{d}p_i} = y_i + \left(p_i - \mu_i - c' - \frac{\partial Z}{\partial y_i}\right)\frac{\partial y_i}{\partial p_i} + \sum_{j=1}^{N}\left(p_j - \mu_j - c' - \frac{\partial Z}{\partial y_j}\right)\frac{\partial y_j}{\partial q}\frac{\mathrm{d}q}{\mathrm{d}p_i}, \tag{A.13}$$

$$\frac{\mathrm{d}q}{\mathrm{d}p_i} = \frac{q_\sigma\left[\sum_{j=1}^{N} 2y_j\sigma_{ij} + \rho\right]\frac{\partial y_i}{\partial p_i} + q_S\frac{\mathrm{d}S}{\mathrm{d}p_i}}{1 - q_\sigma\left[2\sum\sum y_j\frac{\partial y_k}{\partial q}\sigma_{kj} + \sum\frac{\partial y_j}{\partial q}\rho\right]}. \tag{A.14}$$

Substituting and simplifying yield

$$\frac{\mathrm{d}S}{\mathrm{d}p_i} = \frac{y_i + \left(p_i - \mu_i - c' - \frac{\partial Z}{\partial y_i}\right)\frac{\partial y_i}{\partial p_i} + \frac{\sum\left(p_j - \mu_j - c' - \frac{\partial Z}{\partial y_j}\right)\frac{\partial y_j}{\partial q}q_\sigma\left[\sum_{j=1}^{N} 2y_j\sigma_{ij} + \rho\right]\frac{\partial y_i}{\partial p_i}}{V_1}}{V_2}, \tag{A.15}$$

where

$$V_1 = 1 - q_\sigma\left[2\sum\sum y_j\frac{\partial y_k}{\partial q}\sigma_{kj} + \sum\frac{\partial y_j}{\partial q}\rho\right] \tag{A.16}$$

and

$$V_2 = 1 + Z_S - \frac{\sum\left(p_j - \mu_j - c' - \frac{\partial Z}{\partial y_j}\right)\frac{\partial y_j}{\partial q}q_S}{1 - q_\sigma\left[2\sum\sum y_j\frac{\partial y_k}{\partial q}\sigma_{kj} + \sum\frac{\partial y_j}{\partial q}\rho\right]}. \tag{A.17}$$

We thus obtain

$$J_1\left[p_i\left(1 + \frac{1}{\varepsilon_i}\right) - \mu_i - c' - \frac{\partial Z}{\partial y_i}\right] + 2J_2\left[\sum_{j=1}^{N} y_j\sigma_{ij} + \frac{\rho}{2}\right] = 0. \tag{A.18}$$

Multiplying through by y_i and summing over i yield

$$J_1\left[\sum_i y_i\left(p_i\left(1+\frac{1}{\varepsilon_i}\right)-\mu_i-c'-\frac{\partial Z}{\partial y_i}\right)\right]+2J_2\left[\sum_i\sum_j y_iy_j\sigma_{ij}+\sum_i y_i\frac{\rho}{2}\right]=0. \tag{A.19}$$

The proof proceeds in a manner analogous to the previous one if it can be shown that $J_1 \neq 0$. Note that

$$J_1 = 1+\left[\frac{Q_1q_S}{V_1}+(T_S-Z_S)\right]\left(\frac{1}{V_2}\right), \tag{A.20}$$

$$J_2 = T_\sigma+\left[\frac{Q_1q_S}{V_1}+(T_S-Z_S)\right]\left(\frac{1}{V_2}\right)\frac{\sum\left(p_j-\mu_j-c'-\frac{\partial Z}{\partial y_j}\right)\frac{\partial y_j}{\partial q}q_\sigma}{V_1}+\frac{Q_1q_\sigma}{V_1}, \tag{A.21}$$

where

$$Q_1=\sum\left(p_j-\mu_j-c'-\frac{\partial Z}{\partial y_j}\right)\frac{\partial y_j}{\partial q}+T_\sigma\left[2\sum\sum y_j\frac{\partial y_k}{\partial q}\sigma_{kj}+\sum\frac{\partial y_j}{\partial q}\rho\right]. \tag{A.22}$$

If $J_1=0$, Eq. (A.19) implies that J_2 must also be zero. But

$$J_2=0 \Rightarrow T_\sigma-\frac{\left[\sum\left(p_j-\mu_j-c'-\frac{\partial Z}{\partial y_j}\right)\frac{\partial y_j}{\partial q}-Q_1\right]q_\sigma}{V_1}=0. \tag{A.23}$$

Simplifying yields

$$T_\sigma\left[1+\frac{q_\sigma\left[2\sum\sum y_j\frac{\partial y_k}{\partial q}\sigma_{kj}+\sum\frac{\partial y_j}{\partial q}\rho\right]}{1-q_\sigma\left[2\sum\sum y_j\frac{\partial y_k}{\partial q}\sigma_{kj}+\sum\frac{\partial y_j}{\partial q}\rho\right]}\right] \Rightarrow J_2 \neq 0, \tag{A.24}$$

a contradiction. We must again consider two cases:

Case 1: $J_1 \neq 0, J_2 \neq 0$.

This implies that

$$\sum_i y_i\left(p_i\left(1+\frac{1}{\varepsilon_i}\right)-\mu_i-c'-\frac{\partial Z}{\partial y_i}\right)\neq 0, \tag{A.25}$$

which means that we can use Eqs. (A.18) and (A.19) to obtain

$$\frac{p_i\left(1+\frac{1}{\varepsilon_i}\right)-\mu_i-c'-\frac{\partial Z}{\partial y_i}}{\sum_i y_i\left(p_i\left(1+\frac{1}{\varepsilon_i}\right)-\mu_i-c'-\frac{\partial Z}{\partial y_i}\right)}=\frac{\sum_{j=1}^N y_j\sigma_{ij}+\frac{\rho}{2}}{\sum_i\sum_j y_iy_j\sigma_{ij}+\sum_i y_i\frac{\rho}{2}}. \tag{A.26}$$

With the assumption that $\rho/2\sum y_i=0$, multiplying through by $\sum y_i$ and simplifying yield the desired results.

Case 2: $J_1 \neq 0, J_2 = 0$.

The result holds trivially from Eq. (A.18), as $p_i(1 + 1/\varepsilon_i) - \mu_i - c' - \partial Z/\partial y_i = 0$, for all i.

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