The Underinvestment Problem, Bond Covenants and Insurance

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Notation and Assumptions

- s denotes a state of nature and $[0, \bar{s}]$ denote the set of states at t=1;
- Property losses are sustained over the interval $[0,s^l)$, whereas no losses occur over $[s^l, \bar{s}]$.
- $p(s)$ denote the price of a financial asset that pays one dollar if state s occurs and zero otherwise;
- $I(s)$ = investment in state s required to reconstitute the asset;
- $L(s)$ the property loss in state s.
- $L(s) > I(s) > 0$ for $s \in [0,s^l)$; i.e., rebuilding is assumed to always have a positive value.
- The investment decision is made after state s is revealed.

The Underinvestment Problem

- Suppose the firm issued bonds at t=0 with a promised payment of B^U dollars.
- Without covenants, underinvestment occurs in $U \in [0, s^u)$, since bondholders appropriate benefits while shareholders bear costs.
  - The boundary s^u is implicitly defined by the condition $\Pi - I(s^u) = B^u$.
  - Underinvestment reduces firm value by the risk-adjusted present value of the underinvestment cost.
  - Myopic bondholders absorb this loss in corporate value in its entirety.
  - Rational bondholders anticipate the underinvestment incentive when they price the bonds at t=0.

The Underinvestment Problem

The current market value of the debt issue is D^u, where

$$D^u = \int_0^{s^u} p(s)\left[\Pi - L(s)\right] ds + \int_{s^u}^{\bar{s}} p(s)\, B^u\, ds. \tag{1}$$

Similarly, the stock market value S^u of the current shareholders is

$$S^u = \int_{s^u}^{s^l} p(s)\left[\Pi - I(s) - B^u\right] ds + \int_{s^l}^{\bar{s}} p(s)\,[\Pi - B^u]\, ds, \tag{2}$$

and the corporate value, given the underinvestment problem, is V^u, where

$$\begin{aligned} V^u &\equiv D^u + S^u \\ &= \int_0^{s^u} p(s)\,[\Pi - L(s)]\, ds + \int_{s^u}^{s^l} p(s)\,[\Pi - I(s)]\, ds + \int_{s^l}^{\bar{s}} p(s)\,\Pi\, ds. \end{aligned} \tag{3}$$

The Underinvestment Problem

Let V^i denote the corporate value given a certain investment to reconstitute the asset. Then

$$V^i = \int_0^{s^l} p(s)\,[\Pi - I(s)]\,ds + \int_{s^l}^{\bar{s}} p(s)\,\Pi\,ds,$$

and the agency cost is $c^u = V^i - V^u = \int_0^{s^u} p(s)\left[L(s) - I(s)\right] ds > 0.$ (4)

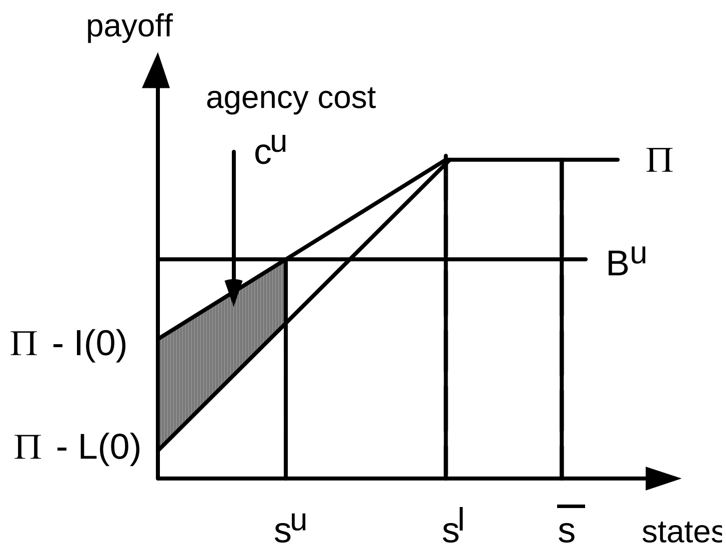

Figure 1: The Agency Cost

Insurance Covenants with No Premium Loading

By purchasing an insurance policy with a deductible of $d = I(s^u)$ dollars, reinvestment is ensured. The premium p^i is given by (5):

$$p^i = \int_0^{s^u} p(s)\,[I(s) - I(s^u)]\,ds. \qquad (5)$$

The (insured) payoff to shareholders is given by (6):

$$\Pi - I(s) + \max\{0, I(s) - I(s^u)\} - B = \begin{cases} 0 & s \in U \\ \Pi - I(s) - B & s \in N \end{cases}. \qquad (6)$$

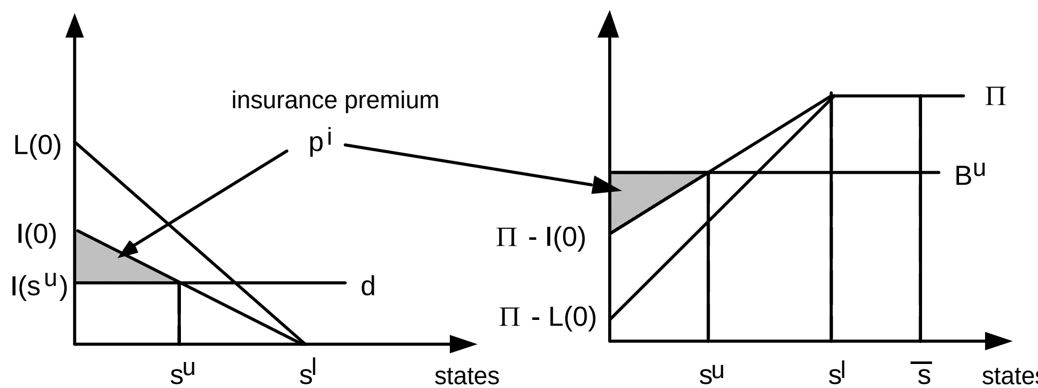

Figure 2: The Insurance Premium

Insurance Covenants with No Premium Loading

By insuring against bankruptcy, the debt becomes safe; therefore,

$$D^i = \int_0^{\overline{s}} p(s)\, B^u\, ds = \rho\, B^u, \tag{7}$$

where ρ is the sum of the basis stock prices, or equivalently, the price now of a safe asset which pays \$1 at $t = 1$. Comparing the insured with the uninsured debt value, the difference represents the sum of the underinvestment cost and the cost of the insurance policy; i.e.,

$$\begin{aligned} D^i - D^u &= \int_0^{s^u} p(s)\, [B^u - (\Pi - L(s))]\, ds \\ &= \int_0^{s^u} p(s)\, [(\Pi - I(s^u)) - (\Pi - L(s))]\, ds \\ &= \int_0^{s^u} p(s)\, \left\{ L(s) - I(s) + [I(s) - I(s^u)] \right\} ds \\ &= c^u + p^i. \end{aligned} \tag{8}$$

Insurance Covenants with No Premium Loading

We endogenize decisions concerning the level of debt (B^c) and insurance deductible ($I(s^c)$). B^c is characterized in the following manner:

$$B^c = \begin{cases} \Pi - I(s) + [I(s) - I(s^c)] & 0 < s < s^c \\ \\ B^c & s^c < s < \overline{s} \end{cases}.$$

B^c is implicitly defined by the financing condition given by (9):

$$D(B^u) - [D(B^c) - p^c] = 0. \qquad (9)$$

Insurance Covenants with No Premium Loading

Let the state s^d be implicitly defined by the condition Π - $L(s^d) = B^c$. Then equation (9) may be equivalently expressed as (10):

$$\int_{\min\{s^d,\ s^u\}}^{s^u} p(s)\left[\Pi - L(s) - B^c\right] ds + \int_{s^u}^{\bar{s}} p(s)\left[B^u - B^c\right] ds$$

$$= \int_0^{\min\{s^d,\ s^u\}} p(s)\left[\min\left\{\Pi - I(s),\ B^c\right\} - \left(\Pi - L(s)\right)\right] ds. \qquad (10)$$

This is shown graphically by Figure 3:

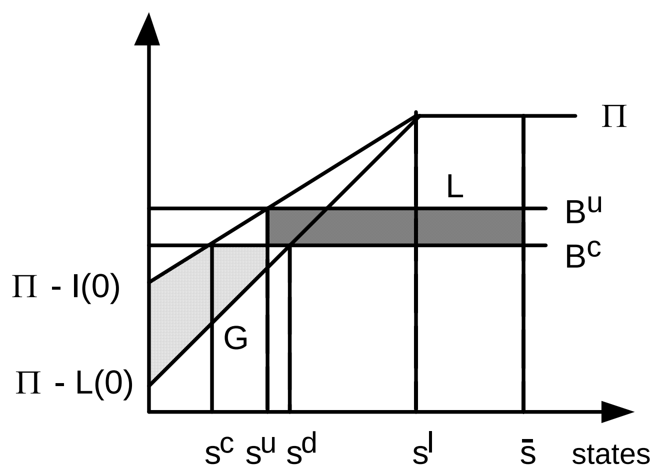

Figure 3: The Financing Condition

Insurance Covenants with No Premium Loading

A bond/covenant scheme is only beneficial if the current shareholders stand to gain. Note the t=1 payoff to current shareholders is

$$\begin{cases} 0 & 0 < s < s^c \\ \Pi - I(s) - B^c & s^c < s < s^l \\ \Pi - B^c & s^l < s < \overline{s} \end{cases}.$$

It follows that the current shareholders' stock market value is S^c, where

$$S^c = \int_{s^c}^{s^l} p(s)\,[\Pi - I(s) - B^c]\,ds + \int_{s^l}^{\overline{s}} p(s)\,[\Pi - B^c]\,ds. \qquad (11)$$

Insurance Covenants with No Premium Loading

$$S^c - S^u = \int_{s^c}^{s^u} p(s)\,[\Pi - I(s) - B^c]\,ds + \int_{s^u}^{\bar{s}} p(s)[B^u - B^c]\,ds$$

$$= \int_{s^c}^{s^u} p(s)\left[\Pi - I(s) - \max\left\{\Pi - L(s),\, B^c\right\}\right] ds$$

$$+ \int_{\min\{s^d,\, s^u\}}^{s^u} p(s)\left[\left(\Pi - L(s)\right) - B^c\right] ds + \int_{s^u}^{\bar{s}} p(s)\left[B^u - B^c\right] ds$$

$$= \int_{s^c}^{s^u} p(s)\left[\Pi - I(s) - \max\left\{\Pi - L(s),\, B^c\right\}\right] ds$$

$$+ \int_{0}^{\min\{s^d,\, s^u\}} p(s)\left[\min\left\{\Pi - I(s), B^c\right\} - \left(\Pi - L(s)\right)\right] ds \qquad (12)$$

$$= \int_{0}^{s^u} p(s)\left[L(s) - I(s)\right] ds = c^u > 0.$$

Insurance Covenants with No Premium Loading

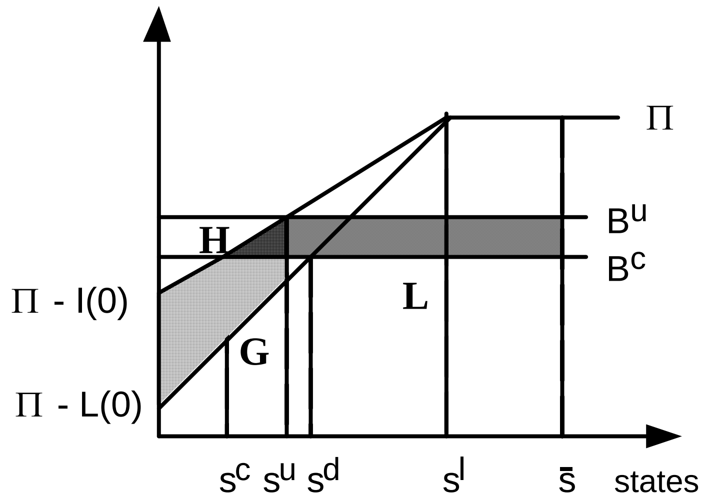

Figure 4

- The difference $S^c - S^u$ is the value of the sum of the shaded areas labeled L and H in Figure 4.
- The value of the sum of the shaded areas labeled G and H represent the agency cost of the underinvestment problem c^u.
- By equation (10) and Figure 3, it follows that the value of the sum of the L and H areas is equal to the value of value of the sum of the G and H areas. Therefore, the bond\covenant package not only increases current shareholder value, but also entirely eliminates the agency cost of underinvestment.

Numerical Example

Table 1

The Unlevered, Uninsured Firm

State	Pr(s)	Π	L(s)	Vu(s) = Π -L(s)	I(s)	Vr(s) = Π-I(s)
no loss	50%	$1000	$0	$1000	$0	$1000
loss	50%	$1000	$800	$200	$600	$400
value now		$1000	$400	$600	$300	$700

Numerical Example

Table 2

The Levered, Uninsured Firm (B = $700)

State	Pr(s)	Π	L(s)	Du(s)	Su(s)	I(s)	Dr(s)	Sr(s)
no loss	50%	$1000	$0	$700	$300	$0	$700	$300
loss	50%	$1000	$800	$200	$0	$600	$400	$0
value now		$1000	$400	$450	$150	$300	$550	$150

Numerical Example

Table 3

Levered, Insured Firm (B^c = $500 and d = $500)

State	Pr(s)	Π	L(s)	I(s)	$p^c(s) = I(s)-d$	$Π^* = Π - I(s) + p^c(s)$	$D^c(s)$	$S^c(s)$
no loss	50%	$1,000	$0	$0	$0	$1,000	$500	$500
loss	50%	$1,000	$800	$600	$100	$500	$500	$0
value now		$1,000	$400	$300	$50	$750	$500	$250

Numerical Example

Table 4

Levered, Insured Firm (B^l = \$600 and d = \$400)

State	Pr(s)	Π	L(s)	I(s)	$p^l(s)$ = I(s)-d	$\Pi^* = \Pi - I(s) + p^l(s)$	$D^l(s)$	$S^l(s)$
no loss	50%	\$1,000	\$0	\$0	\$0	\$1,000	\$600	\$400
loss	50%	\$1,000	\$800	\$600	\$200	\$600	\$600	\$0
value now		\$1,000	\$400	\$300	\$100	\$800	\$600	\$200