Markets for Risk Management

Financial Pricing Models (Part 2)

“Financial Pricing of Insurance in the Multiple-Line Insurance Company”

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Overview and Intuition

- This paper provides a theoretical framework for pricing insurance by line in a multiple line insurer subject to default risk.
  - Previous models (e.g., Doherty-Garven (1986)) implicitly assume monoline insurers.
- This paper also provides an empirical framework for testing the theory, finding that prices vary across firms depending upon overall-firm default risk and the concentration of business among subsidiaries.

Assumptions

- At the beginning of the period, policyholders pay premiums P_1 and P_2 and equityholders input surplus G. These cash flows are invested in separate accounts and evolve over time as (correlated) geometric Brownian motion processes.
- Claims costs (L) are paid at the end of the period.

Equation (1)

Premium Account GBM: $dP_i = \mu_{P_i} P_i dt + \sigma_{P_i} P_i dz_{P_i}$

Surplus Account GBM: $dG = \mu_G G dt + \sigma_G G dz_G$

where $\mu_{P_i}, \mu_G, \sigma_{P_i}$, and σ_G are the drift and volatility parameters for the premium accounts and surplus account, dz_{P_i} and dz_G are Wiener processes, $dz_{P_1} dz_{P_2} = \rho_{P_1 P_2} dt$, and $dz_{P_i} dz_G = \rho_{P_i G} dt$.

Equation (2)

The market value of the firm's line *i* liabilities (not the claims costs themselves) at date τ, $L_i(\tau)$, also evolves according to the following GBM equation:

$$dL_i = \mu_{L_i} L_i dt + \sigma_{L_i} L_i dz_{L_i},$$

where μ_{L_i} and σ_{L_i} are the drift and volatility parameters for the i^{th} liability process, $dz_{L_1} dz_{L_2} = \rho_{L_1 L_2} dt$, $dz_{L_i} dz_G = \rho_{L_i G} dt$, and $dz_{P_i} dz_{L_j} = \rho_{P_i L_j} dt$ for all *i* and *j*.

Equation (3) – Drift Parameters

$$\mu_{P_i} = r_f + \pi_{P_i}, \text{ for premium accounts } i = 1,2$$

$$\mu_G = r_f + \pi_G, \textit{ and} \tag{3}$$

$$\mu_{L_i} = r_{L_i} + \pi_{L_i} \text{ for liability classes } i = 1,2$$

Where

r_f = risk free rate of return,

r_{L_i} = inflation rate for liability class i, i=1,2, and

π_j = the market risk premium for process j=P_i, L_i, and G, i=1,2.

Equation (4) – Risk Premiums

The market risk premium, π_j, is the risk charge investors demand for bearing undiversifiable risk. According to the ICAPM assumption, π_j equals

$$\pi_j = \rho_{jm}\left(\frac{\sigma_j}{\sigma_m}\right)(\mu_m - r_f) \tag{4}$$

where μ_m, σ_m are the drift and diffusion parameters of the Brownian motion process for the market portfolio, respectively, and ρ_{jm} is the instantaneous correlation coefficient between the Brownian motion process for the market portfolio and asset class j, where $j=P_i$, L_i, and G, and i=1,2.

Insurance Pricing with Unlimited Liability

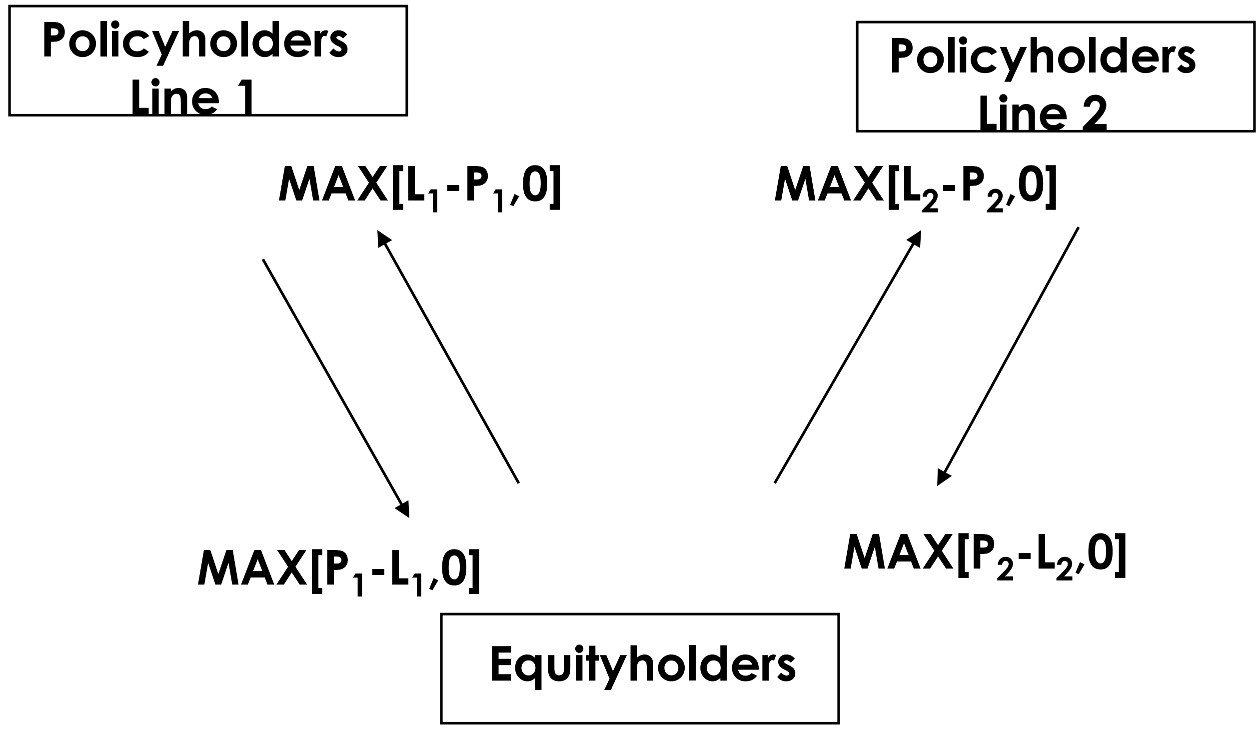
Equation (5) provides the initial market value of the policyholders' claim on an unlimited liability insurer:

$$PH_i(1) = P_i(1) - EH_i(1), \qquad (5)$$

where

$PH_i(1)$ = market value of policyholders' claim;
$P_i(1)$ = value of the premium account; and
$EH_i(1)$ = value of the equityholders' claim on division i.

Multiple-Line Insurer Payoffs



Insurance Pricing with Unlimited Liability

$$C_i(P_i,L_i,\tau) = V_\tau(\text{MAX}[P_i\text{-}L_i,0]), \text{ and}$$
$$B_i(P_i,L_i,\tau) = V_\tau(\text{MAX}[L_i\text{-}P_i,0]).$$

$C_i(P_i,L_i,\tau)$ represents the date τ value of the i^{th} divisional call option, and $B_i(P_i,L_i,\tau)$ represents the date τ value of the i^{th} divisional put option; thus

$$EH_i(\tau) = C_i(P_i,L_i,\tau) - B_i(P_i,L_i,\tau). \quad (6)$$

Substituting (6) into (5) yields (7):

$$PH_i(\tau) = P_i(\tau) - \left[C_i(P_i,L_i,\tau) - B_i(P_i,L_i,\tau)\right]. \quad (7)$$

Insurance Pricing with Unlimited Liability

The put-call parity theorem implies that:

$$\underbrace{C_i(P_i, L_i, \tau)}_{\text{"}call\text{"}} - \underbrace{B_i(P_i, L_i, \tau)}_{\text{"}put\text{"}} = \underbrace{P_i(\tau)}_{\text{"}share\text{"}} - \underbrace{L_i e^{-(r_f - r_{\mathrm{L}_i})\tau}}_{\text{"}bond\text{"}}. \quad (8)$$

Thus $PH_i(\tau) = L_i e^{-(r_f - r_{\mathrm{L}_i})\tau}$; i.e., the fair premium equals the market value of claims generated by line *i*, where the discount rate is the difference between the risk-free rate and the liability growth rate.

Insurance Pricing with Unlimited Liability

Rearranging $PH_i(\tau) = L_i e^{-(r_f - r_{L_i})\tau}$ yields (10):

$$r_{d_i} = -\frac{1}{\tau}\ln\left(\frac{PH_i(\tau)}{L_i}\right) = r_f - r_{L_i} \qquad (10)$$

where r_{d_i} = discount rate for type *i* claims.

Insurance Pricing with Limited Liability

If bankruptcy occurs, policyholder claims are settled on an equal priority basis; i.e., insurer's assets are allocated according to the value of the liability claims held by policyholders. Thus,

$$EH_i(\tau) = C_i(P_i, L_i, \tau) - B_i(P_i, L_i, \tau) + w_{L_i} I(A, L, \tau), \quad (12)$$

where $I(A, L, \tau)$ represents the value of the insolvency put option and A represents the value of the insurer's assets. This implies that

$$PH_i(\tau) = P_i(\tau) - [C_i(P_i, L_i, \tau) - B_i(P_i, L_i, \tau) + w_{L_i} I(A, L, \tau)]; \quad (13)$$

Consequently, $PH_i(\tau) = L_i e^{-(r_f - r_{L_i})\tau} - w_{L_i} I(A, L, \tau). \quad (14)$

Observations and Caveats

- PCA make the following observations:
  - "allocation of surplus to a particular line of business implies that specific lines of business do not have access to the equity capital supporting other lines";
  - "The insurer's equity capital … is available to any line of business where it is needed".
- Caveats: guarantee funds and insurance groups

Insurance Guaranty Funds

- IGF's are funds through which solvent insurers pay assessments to insure payments to persons who have claims against an insolvent company.
- In the United States, all 50 states have guaranty funds for property-casualty insurance; and most states have guarantee funds for life and health insurance.
- Each state sets limits on kinds of policies and amounts of coverage.

Insurance Guaranty Funds

- Policyholders are residual risk bearers because guarantee funds are incomplete in their coverage;
  - Typical limits are $300,000 per claim,
  - Post insolvency assessments are typically limited to 2% of premium volume (so as to limit the "contagion" effect of insurer insolvencies).
- Taxpayers are also residual risk bearers, since insurers who pay into guarantee funds receive credits on premium taxes.

Insurance Group vs. Unitary Insurer

Insurance Group

Subsidiary 1:	Subsidiary 2:	Subsidiary 3:
Assets A1	Assets A2	Assets A3
Liabilities L1	Liabilities L2	Liabilities L3
Price1 = PV(L1)-Put(A1,L1)	Price2 = PV(L2)-Put(A2,L2)	Price3 = PV(L3)-Put(A3,L3)

Unitary Insurer

Assets A = A1+A2+A3
Liabilities L = L1+L2+L3
Price(U) = PV(L)-Put(A,L)

Price(U) > Price1 + Price2 + Price3

Empirical results

- Price is inversely related to firm risk for long tailed lines.
- Price is less sensitive to firm risk for short-tail lines.
- Price is positively related to group concentration.

Implications for Capital Allocation

- PCA suggest that it is not appropriate to allocate capital by line of business (since the equal priority rule implies that the price of insurance by line is determined by the overall default risk of the firm).
- Myers-Read take issue with PCA on this point by showing that the marginal contribution to default risk in fact varies across lines of business!