# Markets for Risk Management

## Reinsurance, Taxes and Efficiency: A Contingent Claims Model of Insurance Market Equilibrium

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# Motivation

- We examine the role played by corporate income taxation in creating incentives for firms to contractually reallocate risk (in this case, via reinsurance).
- Historically, the prospect of tax shield under-utilization has been an important problem.
- Modeling considerations – there must be a "cost" associated with low income; although insurers are risk neutral, the valuation function is nonlinear due to the convexity of the firm's tax liability.

# Other Approaches

- Expected utility framework (Borch (1960, 1962)) -- with HARA utility, reinsurance supplied and demanded on a proportional basis.
- Mean-variance framework (Blazenko (1986), Eden and Kahane (1990))
- Value Maximization framework (Doherty/Tiniç (1981), Garven (1987), Garven and Lamm-Tennant (2003), Mayers/Smith (1982, 1990)

# Contributions

- Analysis of the effect of taxes on underwriting capacity and equilibrium in insurance and reinsurance markets
- Asymmetric taxes 1) cause reinsurance to yield net tax benefits, and 2) are <u>sufficient</u> (although not <u>necessary</u>) for the existence of reinsurance.
- In equilibrium, asymmetric taxes cause insurance prices to be actuarially unfair, and the expected return on capital invested in insurance reflects the probability of paying taxes.
- More generally, asymmetric taxes create a corporate demand for hedging (irrespective of investor risk preferences).

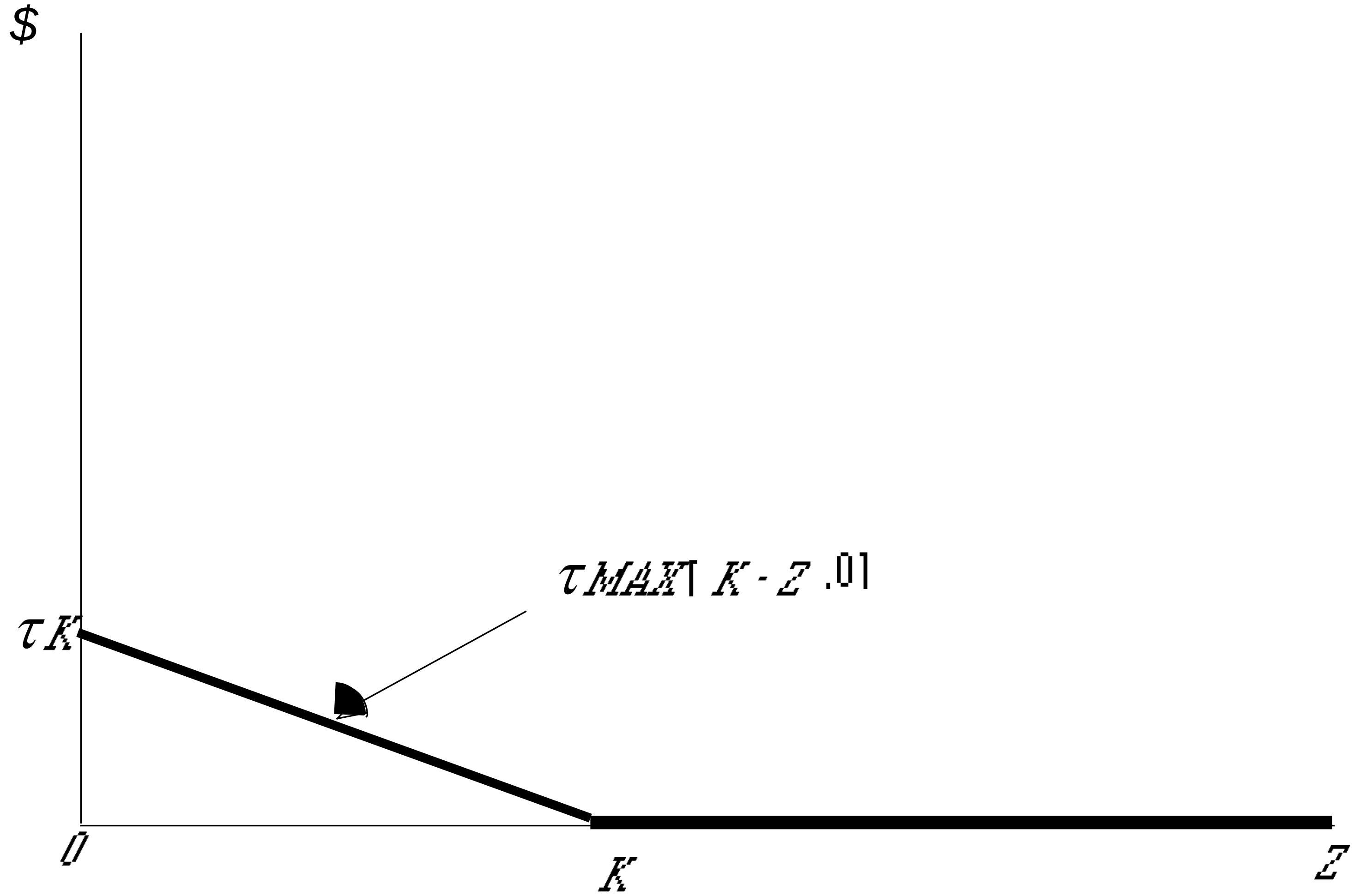
# Model Assumptions and Notation

- Two periods: $t=0$, the present, and $t=1$, the future;
- No contracting costs or bankruptcy risk;
- Insurer assets are riskless;
- Aggregate claims costs $X \sim N(E_x, \sigma_x)$ and are stochastically independent of social wealth;

# Model Assumptions and Notation

- Competitively structured insurance market, where $p$ represents the aggregate premium of the economy-wide risk pool;
- $m$ risk neutral insurers differ with respect to endowed surplus $S_j$. These insurers must optimally select a proportionate share $\gamma_j$ ($\gamma_j \in [0,1]$) of the economy-wide risk pool. Initially, $m$ and $S_j$ are assumed to be fixed for all $j$;
- Asymmetric tax regime levies taxes on the sum of underwriting profit and investment income at the rate $\tau$; however, losses are not rebated.

# **Figure 1:** Profile of Tax Payment



# The Model

$$V(\gamma) = S + \gamma p - R^{-1}E_z - \tau R^{-1}E\{Max[0, K-Z]\}, \qquad (1)$$

where $R = 1 + r$, $K = rS + \gamma pR$; and $Z = \gamma x$. Let $P(K,Z) = R^{-1}E\{Max[0,K-Z]\}$. Then

$$V(\gamma) = S + \gamma(p - R^{-1}E_x) - \tau P(K,Z). \qquad (1')$$

Since $Z \sim N(E_z, \sigma_z)$,

$$P(K,Z) = R^{-1}\gamma\sigma_x[dN(d) + n(d)], \qquad (2)$$

where

$$d = \frac{K - E_z}{\sigma_z} = \frac{rS + \gamma(pR - E_x)}{\gamma\sigma_x}. \qquad (3)$$

# The Model

$$\frac{\partial P}{\partial \gamma} = R^{-1}\boldsymbol{\sigma}_x[\mathrm{N}(d)(d + \gamma\frac{\partial d}{\partial \gamma}) + \mathrm{n}(d)] \qquad (4)$$

$$= R^{-1}[\mathrm{N}(d)(pR - \mathrm{E}_x) + \boldsymbol{\sigma}_x \mathrm{n}(d)].$$

$$\frac{\partial^2 P}{\partial \gamma^2} = -\frac{rS}{\gamma} R^{-1}\mathrm{n}(d)\frac{\partial d}{\partial \gamma} > 0. \qquad (5)$$

From equations (1') and (5) we obtain the second-order condition (*SOC*):

$$V''(\gamma) = -\tau(\partial^2 P / \partial \gamma^2) < 0.$$

# The Model

From equations (1') and (4) we obtain the first-order condition (*FOC*):

$$\underbrace{(pR - E_x)[1 - \tau N(d)]}_{\text{after-tax marginal underwriting gain}} = \underbrace{\tau\sigma_x n(d)}_{\text{marginal tax loss}} \qquad (6)$$

Solving (6) for $p$:

$$p = R^{-1}\{E_x + \lambda(\gamma)\sigma_x\}, \qquad (7)$$

where $\lambda(\gamma) = \dfrac{\tau\, n(d)}{1 - \tau\, N(d)} \geq 0$ represents the unit risk loading factor; i.e., the loading required per dollar of expected loss that compensates the insurer for the tax burden of underwriting risk.

# The Model

- From (7), it is apparent that insurance must be actuarially unfair; otherwise, the optimal value for $\gamma$ is zero.
  - To see this, suppose insurance is actuarially fair (i.e., $p = R^{-1}E_x$). Then $n(d) = 0$, and $d \rightarrow \infty$. Since $d = rS/\gamma\sigma_x$, $\gamma = 0$; hence, no insurance is supplied.
- For positive values of $\gamma$, the *FOC* is satisfied <u>only</u> if insurance is actuarially unfair (i.e., if $p > R^{-1}E_x$).
- If there are no taxes; i.e., if $\tau = 0$, then insurance prices are actuarially fair.

# Brief Tutorial: Implicit Function Theorem

- The implicit function theorem states that given some function $F(y, x_1, ..., x_n) = 0$, if an implicit function $y = f(x_1, ..., x_n)$ exists, then the partial derivatives of the implicit function are $\frac{\partial y}{\partial x_i} = -\frac{\partial F / \partial x_i}{\partial F / \partial y}$, for all $i$, $i$ = 1,...,$n$.

- The first-order condition for the present model is $V_\gamma(\gamma^*, x_1, ..., x_n)$=0, where $V_\gamma$ corresponds to the partial derivative of equity value with respect to $\gamma$, the $x_i$'s represent model parameters (i.e., $p$, $E_X$, $r$, $\tau$, $\sigma_X$ and $S$), and $\gamma^* = f(x_1, ..., x_n)$ is the implicit function. Therefore, $\frac{\partial \gamma^*}{\partial x_i} = -\frac{\partial V_\gamma / \partial x_i}{\partial V_\gamma / \partial \gamma}$ for all $i$, $i$ = 1,...,$n$.

- Since $\partial V_\gamma / \partial \gamma = \partial^2 V / \partial \gamma^2 < 0$, this implies that the sign of $\partial \gamma^* / \partial x_i$ will be the same as the sign of $\partial^2 V / \partial \gamma \partial x_i$; e.g., sign($\frac{\partial \gamma^*}{\partial S} = \frac{\partial V^2}{\partial \gamma / \partial S}$).

# The Model

**Lemma 1:** The optimal insurance supply increases in $S$, $p$, and $r$, and decreases in $\tau$ and $\sigma_x$.

PROOF: We show the proof for surplus ($S$) only. Differentiating implicitly from the *FOC* with respect to $\gamma$ and S,

$$\frac{\partial \gamma}{\partial \mathrm{S}} = -\frac{\partial V' / \partial \mathrm{S}}{V''(\gamma)} .$$

Since $V''(\gamma) < 0$, $\mathrm{sign}(\partial \gamma / \partial \mathrm{S}) = \mathrm{sign}(\partial V' / \partial \mathrm{S})$.

$$\partial V' / \partial \mathrm{S} = -\tau(\partial^2 P / \partial \gamma \partial S) = \tau \mathrm{n}(d) r^2 S / R \gamma^2 \sigma_x > 0. \quad (8)$$

Hence $\partial \gamma / \partial S > 0$.

# Market Equilibrium

The market equilibrium condition is written:

$$\sum_j \gamma_j = 1. \qquad (9)$$

- From the *FOC*, the optimal market share $\gamma$ is a function of six parameters: $p$, $E_x$, $r$, $\tau$, $\sigma_x$ and $S$.
- From Lemma 1, using (5) and (8) and the expression for V''($\gamma$), we obtain: $\partial\gamma / \partial S = \gamma / S$; hence $\gamma$ is <u>linear</u> in $S$. Therefore, the optimal market share for insurer $j$, $\gamma_j$, can be written as

$$\gamma_j = S_j \bullet \mathrm{h}(p, \tau, \sigma_x, E_x, r).$$

# Market Equilibrium

Applying the market equilibrium condition, $\sum_j \gamma_j = h(\bullet) \sum_j S_j = 1$. Consequently, $h(\bullet) = 1/\Sigma_j S_j$, and

$$\gamma_j = S_j / \Sigma_j S_j = s_j, \qquad (10)$$

where $s_j$ represents insurer $j$'s share of total industry surplus. Equation (10) indicates an optimal sharing rule that we formally define in the following proposition:

**Proposition 1:** In equilibrium, the share of insurer $j$ in the insurance market is equal to its share in the industry's surplus: $\gamma_j = s_j$.

# Reinsurance and Efficiency

Let $\alpha_j$ represent the $j^{th}$ insurer's endowed market share, and $\beta_j$ represent the fraction of the market which insurer $j$ reinsures ($\beta_j \leq \alpha_j$).

$$\therefore V(\beta) = S + (\alpha - \beta)(p - R^{-1}E_x) - \tau P(K,Z), \qquad (11)$$

where $K = rS + (\alpha - \beta)pR$; and $Z = (\alpha - \beta)X$. From (11), all previous results obtain, where $\gamma$ is simply replaced by $\alpha - \beta$. Furthermore, the optimal sharing rule is

$$\beta_j = \alpha_j - s_j. \qquad (12)$$

# Reinsurance and Efficiency

- Next, consider a special case of (12). Suppose all insurers underwrite the same share of the insurance market; i.e., if $\alpha_j = 1/m$ for all $j$, then we obtain:

$$\beta_j = (1/m) - s_j.$$

Since average surplus $\bar{S} = \Sigma_j S_j / m$ and the average surplus share is $\bar{s} = \bar{S} / \Sigma_j S_j$, $\bar{s} = 1/m$. Hence,

$$\beta_j = \bar{s} - s_j.$$

Thus the proposition:

**Proposition 2:** In equilibrium, given $\alpha$, high surplus firms reinsure low surplus firms.

# International Reinsurance and Taxes

- Reinsurers operating in low-tax domiciles augment underwriting capacity of local insurers in high-tax domiciles.
- Internationally, a significant proportion of reinsurance underwriting capacity is in fact provided by specialist reinsurers (e.g., off-shore captives operating in low-tax domiciles).
- Empirical implication – inverse relationship between tax rates and net retention ratios is confirmed by Outreville (1994) in a cross-sectional study of the relation between retention rates and corporate tax rates in 42 developing countries.

# Surplus and the Equilibrium Rate of Return

- Previously, number of insurers = $m \Rightarrow$ actuarially unfair price for insurance (see (7)).
- What if the number of insurers and amount of surplus are endogenous?
  - Since insurance is unfair, there is an incentive for entry $\Rightarrow$ increase in industry surplus!

# Surplus and the Equilibrium Rate of Return

- Let $S^* = \Sigma_j S_j$ represent the total surplus of the insurance industry. In Proposition 1, we noted that $S^*$ enters into the equilibrium expression for *d*:

$$d = \frac{rS^* + pR - E_x}{\sigma_x} \tag{18}$$

Substituting $p = R^{-1}\{E_x + \lambda(\gamma)\sigma_x\}$ (see (7)), we obtain:

$$\frac{pR - E_x}{\sigma_x} = \frac{\tau n(d)}{1 - \tau N(d)} = \lambda(\tau, d) \tag{19}$$

Substituting (19) into (18), we obtain a new equilibrium expression for *d*:

$$d = \frac{rS^*}{\sigma_x} + \lambda(\tau, d). \tag{20}$$

# Surplus and the Equilibrium Rate of Return

- Next, define $E(r_i)$ as the after-tax expected rate of return on capital invested in an insurance firm. By definition,

$$E(r_i) = S^{-1}\{RS + \gamma(pR - E_x) - \tau E[Max(0, K\text{-}Z)] - S\}.$$

- Using (2), (7), (10) and (20), we derive the equilibrium value for $E(r_i)$:

$$E(r_i) = r[1 - \tau N(d)]. \qquad (21)$$

(21) implies that the after-tax expected rate of return on insurance equals the after-tax rate of return on the riskless asset, adjusted for the probability of paying taxes.

- Since $N(d) < 1$, asymmetric taxes ⇒ reward for idiosyncratic risk (equal to the expected tax payment, adjusted for the probability that losses are sustained); i.e., $E(r_i) - r(1\text{-}\tau) = \tau r(1\text{-}N(d))$.

# Surplus and the Equilibrium Rate of Return

- **Lemma 2:** The equilibrium value of $d$ is increasing in $S^*$ (implying that $\mathrm{d}E(r_i)/\mathrm{d}S^* < 0$)

  PROOF: From equation (20), taking $r$, $\tau$ and $\sigma_X$ as given, $d$ is defined by an implicit function:

$$\mathrm{F}(d, S^*) = d - \frac{rS^*}{\sigma_x} - \lambda(\tau, d) = 0.$$

  When $S^*$ increases, the total change in $d$ from one equilibrium state to the other is

$$\frac{\mathrm{d}d}{\mathrm{d}S^*} = -\frac{\partial \mathrm{F}/\partial S^*}{\partial \mathrm{F}/\partial d} = \frac{r}{\sigma_x}\left(1 - \frac{\partial \lambda}{\partial d}\right)^{-1}.$$

  Since $\partial\lambda/\partial d < 0$, $\mathrm{d}d/\mathrm{d}S^* > 0$.

  **Proposition 4:** Long-run equilibrium obtains when $E(r_i) = r(1\text{-}\tau)$; idiosyncratic risk is rewarded by an excess return depending on the tax rate and on the probability that the insurance business generates losses.