Markets for Risk Management

The Demand for Reinsurance: Theory and Empirical Tests

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This Paper's Contributions

- This paper sets forth a contingent claims-based theory of reinsurance demand in which reinsurance is viewed as both a capital management and risk management mechanism.
- Empirical study estimates comparative static relationships derived directly from the contingent claims model.
- Empirical evidence supports most of model predictions; specifically, reinsurance demand is positively related to leverage and claim delays, and negatively related to the correlation between assets and claim costs.

Model Setting

- Single period framework - insurer is formed for the purpose of maximizing the after-tax market value of its equity (surplus).
- Insurer issues a set of insurance policies for which it receives total premium income of P dollars; some proportion of liabilities is reinsured at a cost of P_r dollars.
- Initial surplus (S_0) and premium income net of reinsurance (P_n) are invested in the financial market.
- At t=1, the insurer generates a set of cash flows from its investment, underwriting, and reinsurance activities.
- The insurer's problem is to select the optimal level (α) of reinsurance for the policies it has decided to underwrite.

Model Assumptions

- Perfectly competitive financial markets and insurance markets;
- Insurers are subject to the risk of insolvency;
- Reinsurers are not subject to the risk of insolvency;
- Investors' utility functions exhibit constant absolute risk aversion; and
- Investment returns, claims costs, and terminal wealths are multivariate normally distributed.

Model Notation

- α = quota share reinsurance decision variable; $\alpha \in [0,1]$;
- $\pi(\alpha)$ = default cost function, $\pi'(\alpha) < 0$;
- $P(\alpha) = P - \pi(\alpha)$ = gross premium income, $P'(\alpha) > 0$;
- P_r = price of "full coverage" reinsurance;
- $P_n(\alpha) = P(\alpha) - \alpha P_r$ = net premiums written, $P_n(\alpha) < 0$;
- $A(\alpha) = S_0 + kP_n(\alpha)$ = insurer's initial assets, $A'(\alpha) < 0$;
- k = average claim delay (funds generating coefficient);
- θ = proportion of the insurer's investment income subject to taxation; $\theta \in [0,1]$;

Model Notation

- $f(r_p, L)$ = bivariate normal density function governing the insurer's investment returns (r_p) and claims costs (L);
- $\hat{f}(r_p, L)$ = corresponding risk neutral bivariate density function;
- r_f = riskless rate of interest;
- r_m = rate of return on the market portfolio;
- $R_i = 1 + r_i$, i = f,m,p;
- $n(.)$ = standard normal density function;
- $N(.)$ = cumulative standard normal distribution function.

Pre-Tax Value of the Insurance Firm

$$C(AR_p;-U) = R_f^{-1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} MAX\left[\left(AR_p + P_n - (1-\alpha)L\right),0\right]\hat{f}(r_p,L)\,dr_p\,dL \quad (1)$$

Let $Y = AR_p - (1-\alpha)L$, $\hat{E}(Y) = AR_f - (1-\alpha)\hat{E}(L)$, and $\sigma_Y^2 = A^2\sigma_p^2 + (1-\alpha)^2\sigma_L^2 - 2A(1-\alpha)\sigma_{pL}$. Then

$$C(AR_p;-U) = R_f^{-1}\int_{-P_n}^{\infty}(Y + P_n)\,\hat{f}(Y)\,dY. \quad (2)$$

Pre-Tax Value of the Insurance Firm

Changing the random variate Y to a standardized normal variate y yields

$$C(AR_p;-U)=\left\{A+\left[P_n-(1-\alpha)\hat{E}(L)\right]R_f^{-1}\right\}N(X_1)+R_f^{-1}\sigma_y n(X_1), \quad (3)$$

where

$X_1=\left[AR_f+P_n-(1-\alpha)\hat{E}(L)\right]/\sigma_y$ = standardized risk neutral terminal value of pre-tax profit;

$N(X_1)$ = risk neutral solvency probability (note that the "true" solvency probability is higher than this).

The Value of the Government's Claim

The value of the government's claim, $\tau C(A\theta r_p;-U)$, is:

$$\tau C(A\theta r_p;-U) =$$

$$\tau\, R_f^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} MAX\left[\left(A\theta r_p + P_n - (1-\alpha)L\right), 0\right] \hat{f}(r_p, L)\, dr_p\, dL \qquad (4)$$

Let $Z = A\theta r_p - (1-\alpha)L$, $\hat{E}(Z) = A\theta r_f - (1-\alpha)\hat{E}(L)$, and $\sigma_z^2 = A^2\theta^2\sigma_p^2 + (1-\alpha)^2\sigma_L^2 - 2A\theta(1-\alpha)\sigma_{pL}$. Then

$$\tau\, C(A\theta r_p;-U) = \tau\, R_f^{-1} \int_{-P_n}^{\infty} (Z + P_n)\, \hat{f}(Z)\, dZ. \qquad (5)$$

The Value of the Government's Claim

Changing the random variable Z to a standardized normal variate z yields

$$\tau\, C(A\theta r_p;-U)=\tau\, R_f^{-1}\left\{A\theta r_f+P_n-\left(1-\alpha\right)\hat{E}\left(L\right)\right\}N(X_2)+\tau\, R_f^{-1}\boldsymbol{\sigma}_z n(X_2)$$

(6)

where

$X_2=\left[A\theta r_f+P_n-(1-\alpha)\hat{E}(L)\right]/\boldsymbol{\sigma}_z$ = standardized risk neutral terminal value of taxable profit;

$N(X_2)$ = the risk neutral probability that the insurer will be taxed.

Insurer's Objective Function

$$\max_{\alpha} V_e = \left\{ A + \left[P_n - (1-\alpha)\hat{E}(L) \right] R_f^{-1} \right\} N(X_1) + R_f^{-1} \sigma_y n(X_1)$$

$$- \tau R_f^{-1} \left\{ A\theta r_f + P_n - (1-\alpha)\hat{E}(L) \right\} N(X_2) + \tau R_f^{-1} \sigma_z n(X_2)$$

First-Order Condition

$$\frac{\partial V_e}{\partial \alpha} = \underbrace{-P_r\left[N\left(X_1\right)-\tau R_f^{-1}N\left(X_2\right)\right]}_{\text{after}-\text{tax marginal cost of reinsurance}}$$

$$+\underbrace{\left[\hat{E}(L)R_f^{-1}-\frac{\partial \pi}{\partial \alpha}\right]\left[N\left(X_1\right)-\tau R_f^{-1}N\left(X_2\right)\right]}_{\text{after-tax marginal benefits of lower claims and agency costs}}$$

$$\underbrace{-k\left[\frac{\partial \pi}{\partial \alpha}+P_r\right]\left[N\left(X_1\right)-\theta\tau r_f R_f^{-1}N\left(X_2\right)\right]}_{\text{after-tax marginal cost of foregone investment income}}$$

$$+\underbrace{R_f^{-1}\left[\frac{\partial \sigma_y}{\partial \alpha}n\left(X_1\right)-\tau\frac{\partial \sigma_z}{\partial \alpha}n\left(X_2\right)\right]}_{\text{marginal effects of changes in variability}}=0.$$

Testable Hypotheses

- **Hypothesis 1 (reinsurance/surplus substitutability):** Other things equal, the demand for reinsurance will be greater the higher the insurer's leverage;
- **Hypothesis 2 ("natural hedge" argument):** Other things equal, the demand for reinsurance will be greater the lower the correlation between the insurer's investment returns and claims costs.
- **Hypothesis 3 (longer tail – leverage effect):** Other things equal, the demand for reinsurance will be greater for insurers that write "longer-tail" lines of insurance;
- **Hypothesis 4 (tax effect):** Other things equal, the demand for reinsurance will be greater for insurers that concentrate their investments in tax-favored assets.

Data

- Eight years of data (1980-1987) for 179 insurers were obtained from the A. M. Best database. Sample selection criteria were as follows:
- The insurer must be an unaffiliated single company.
- The insurer must have been classified as either a stock or mutual company during the entire eight-year period, and it cannot be classified as a specialist reinsurer.
- Since a number of variables in the regression model involve ratios, only those insurers reporting positive (nonzero) values for the denominators of these ratios are included in the sample so as to avoid division by zero.

Empirical Model

$$REINS_j = \beta_{0j} + \sum_{i=1}^{17} \beta_{ij} X_{ij} + \varepsilon_{ij}, \text{ where}$$

$REINS_j$ = reinsurance premiums/total business premiums;

$X_{1j} = SIZE_j$ = natural logarithm of admitted assets;

$X_{2j} = PSRATIO_j$ = ratio of direct premiums written/surplus;

$X_{3j} = RHO_j$ = correlation between assets and liabilities;

$X_{4j} = STDP_j$ = standard deviation of investment returns;

$X_{5j} = STDL_j$ = standard deviation of claims costs;

$X_{6j} = SCHEDP_j$ = proportion of premiums written in Schedule P lines;

Empirical Model

X_{7j} = $THETA_j$ = proportion of investment income subject to taxation;

$$X_{8j} = HERF_j = \sum_{i=1}^{n} \left(\frac{(\text{Direct Premiums Written})_{ij}}{\sum_{i=1}^{n} (\text{Direct Premiums Written})_{ij}} \right)^2 ;$$

X_{9j} = $LICENSE_j$ = - number of states in which j is licensed;

X_{10j} = $MUTUAL_j$ = 1 if j is a mutual, 0 if j is a stock company;

X_{11j}-X_{17j} = T_1-T_7 = year indicators; T_1=1 if $YEAR$ = 1981, …, T_7=1 if $YEAR$=1987, 0 otherwise.

Risk Proxies

- 17 liability covariances based up aggregate industry data for 1970-1994;
- 14 asset covariances based upon 300 monthly observations per asset type from Ibbotson and Associates;
- Asset/liability covariances based up 25 annual observations from aggregate industry data and Ibbotson and Associates;
- *t* tests indicate intertemporal stability for all asset covariances and most insurance liability covariances;
- Firm-specific proxies for asset variances, liability variances, and asset-liability covariances based upon applying the following formulas using *X* and *W* values from A. M. Best Balance Sheet and Income Statement database.

Risk Proxy Formulas

$$\rho_{PL}^{j} = RHO_{j} = \sum_{\substack{i=1 \\ i \neq k}}^{14} \sum_{k=1}^{17} X_{ij} W_{kj} COV(r_{ij}, L_{kj}) / \sigma_{pj} \sigma_{L_j};$$

$$\sigma_{P}^{j} = STDP_{j} = \sum_{i=1}^{14} \sum_{k=1}^{14} X_{ij} X_{kj} COV(r_{ij}, r_{kj});$$

$$\sigma_{L}^{j} = STDL_{j} = \sum_{i=1}^{17} \sum_{k=1}^{17} W_{ij} W_{kj} COV(L_{ij}, L_{kj});$$

$$\sigma_{E}^{j} = STDE = \sigma_{P}^{2} + \sigma_{L}^{2} - 2\sigma_{P}\sigma_{L}\rho_{PL}.$$

Table 1: Panel Data Summary Statistics

Variable	Mean	Standard Deviation	Minimum	Maximum
REINS	0.2724	0.2163	-0.0912	1.0012
SIZE	16.7714	1.3024	13.5140	20.7231
PSRATIO	2.4754	1.6136	0.0078	9.9674
HERF	0.4320	0.2164	0.0017	0.9988
RHO	0.1145	0.1268	-0.3729	0.4017
STD_E	0.1519	0.0926	0.0289	0.4925
STD_P	0.0587	0.0254	0.0230	0.1690
STD_L	0.0139	0.0219	0.0000	0.1206
SCHED_P	0.6434	0.2414	0.0000	0.9980
THETA	0.6983	0.2013	0.0303	1.0000
LICENSE	-46.8393	13.9387	-56.0000	-2.0000
MUTUAL	0.5970	0.4907	0.0000	1.0000

Figure 1: Univariate relationship between reinsurance and equity risk

Model Equation

REINS = 0.2300 + 0.2790 STD_E

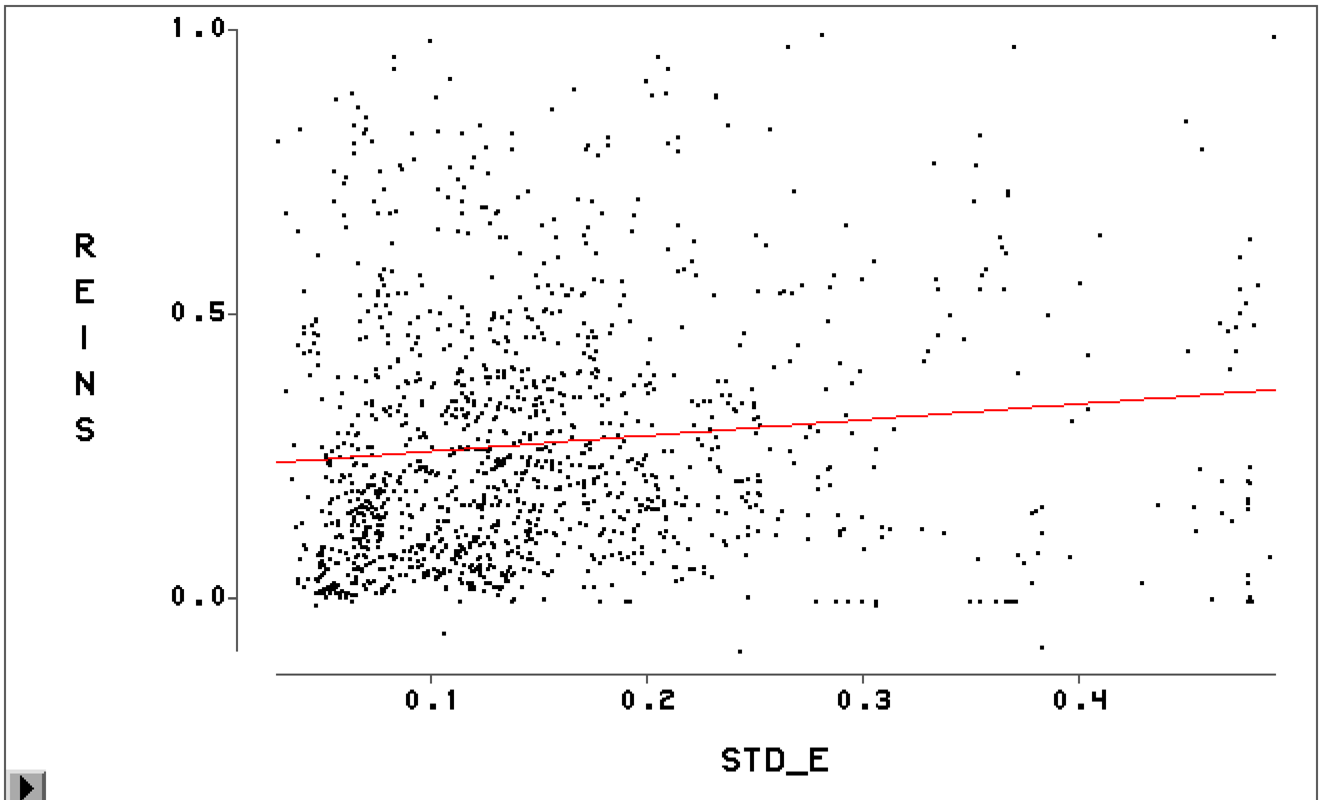


Figure 2: Univariate relationship between reinsurance and leverage

Model Equation				
REINS =	0.1890	+	0.0337	PSRATIO

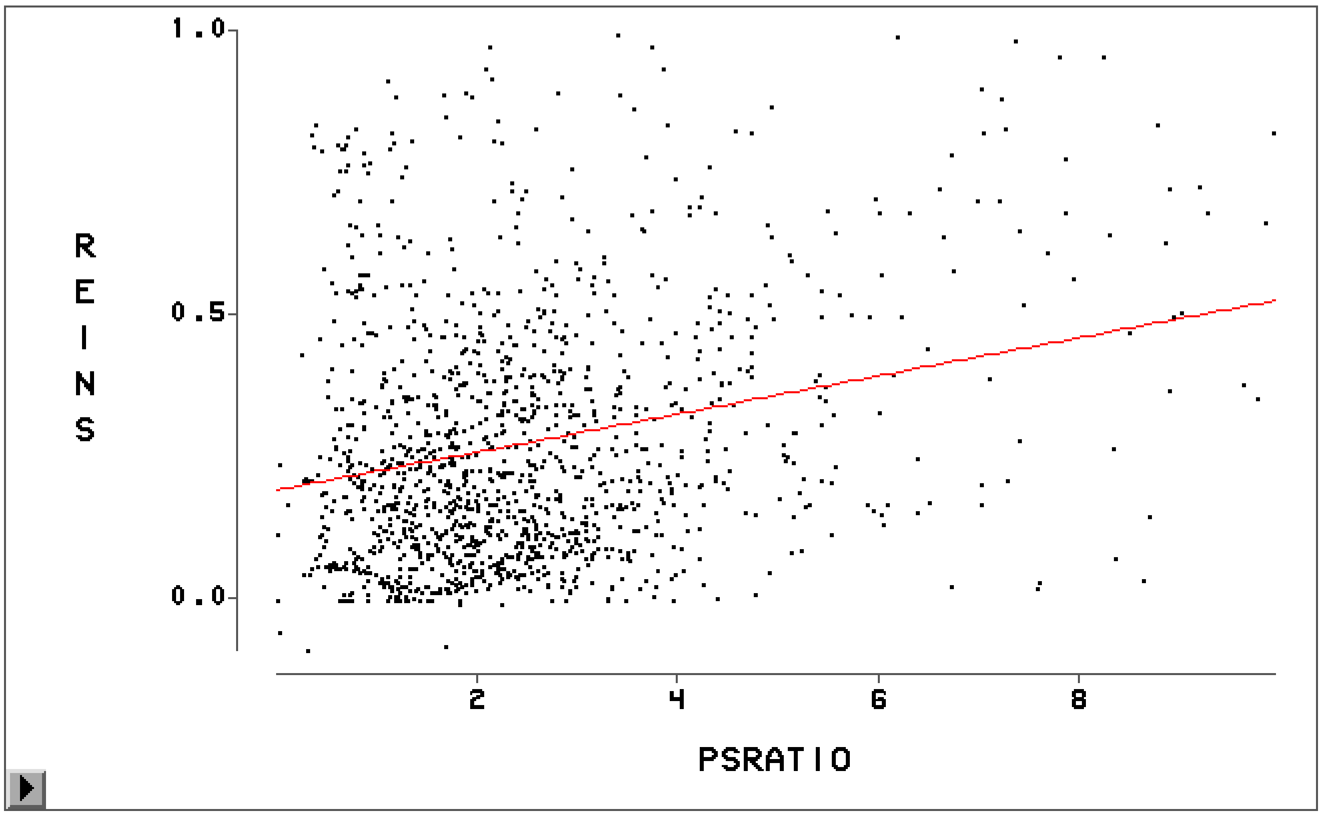


Figure 3: Univariate relationship between reinsurance & asset-liability correlation

Model Equation

REINS = 0.3085 - 0.3147 RHO

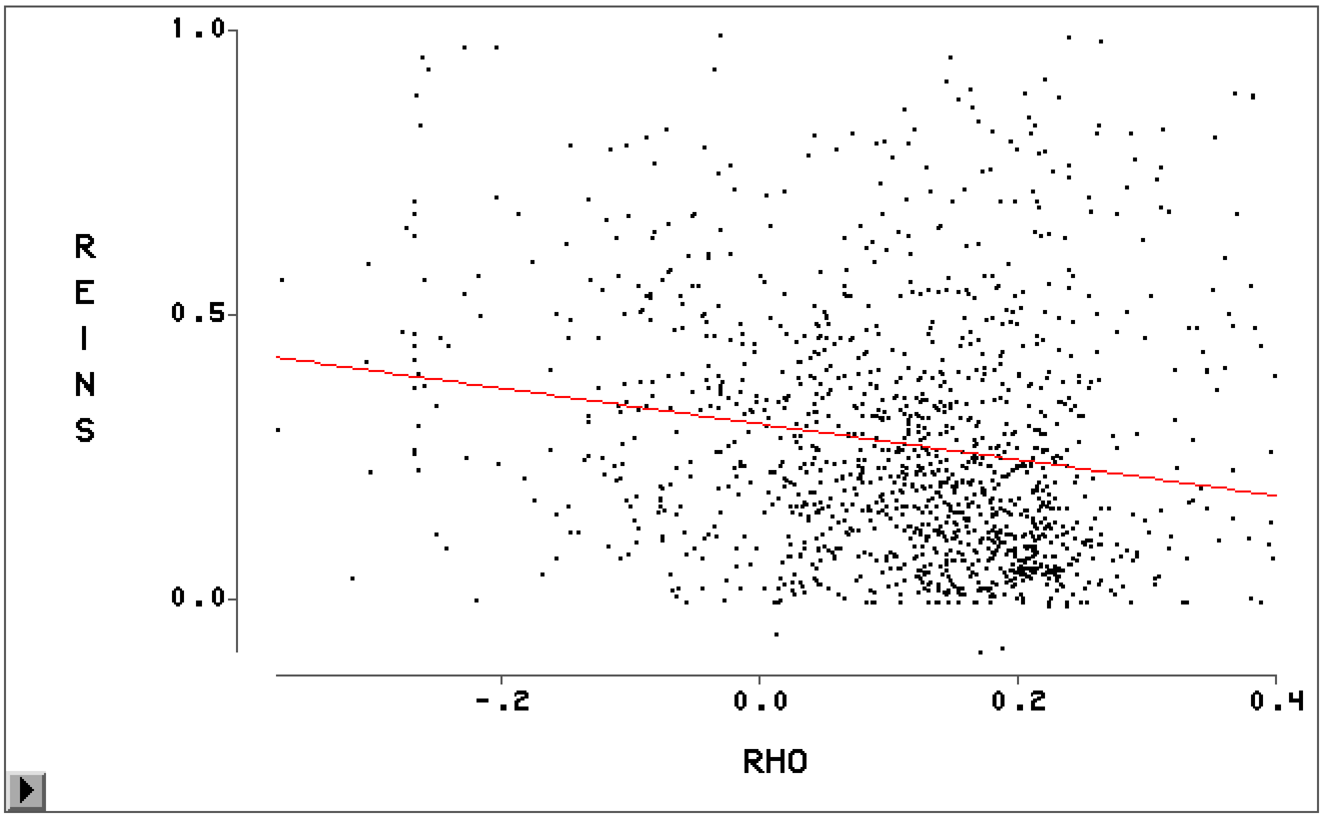
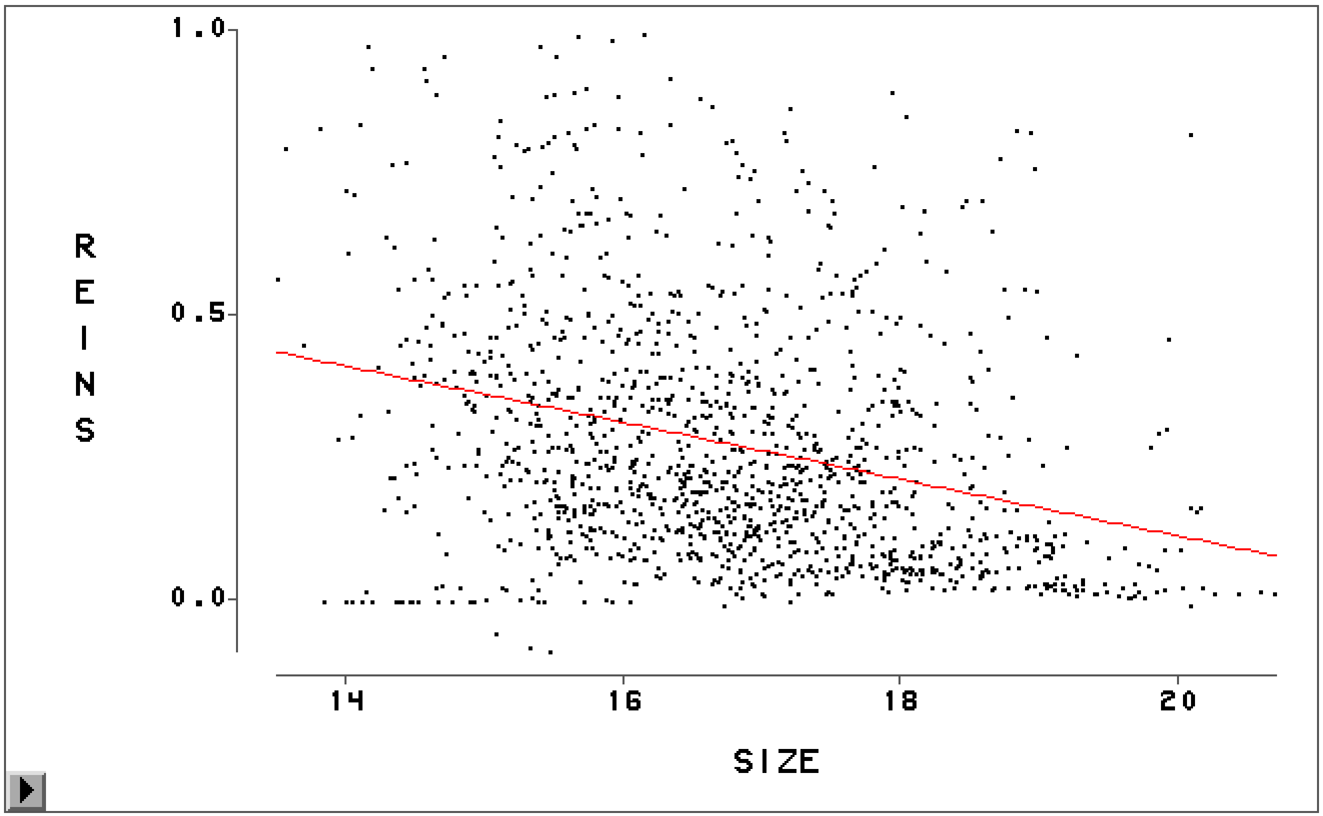


Figure 4. Univariate relationship between reinsurance and firm size

Model Equation

REINS = 1.1097 - 0.0499 SIZE



Multivariate Results

Model: EQ1 (Base Case)
R-square 0.3207

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	**1**	**1.758757**	**0.10460238**	**16.814**	**0.0001**
SIZE	**1**	**-0.077733**	**0.00504193**	**-15.417**	**0.0001**
PSRATIO	**1**	**0.023916**	**0.00323816**	**7.386**	**0.0001**
HERF	**1**	**-0.046094**	**0.02405551**	**-1.916**	**0.0556**
RHO	**1**	**-0.107765**	**0.04618680**	**-2.333**	**0.0198**
STD_P	**1**	**0.894856**	**0.23334461**	**3.835**	**0.0001**
STD_L	1	0.317787	0.32325521	0.983	0.3257
SCHED_P	**1**	**0.098643**	**0.02852486**	**3.458**	**0.0006**
THETA	1	-0.030356	0.02613178	-1.162	0.2456
LICENSE	**1**	**-0.006929**	**0.00042166**	**-16.434**	**0.0001**
MUTUAL	1	-0.004909	0.01126415	-0.436	0.6630
T1	1	0.005653	0.01941708	0.291	0.7710
T2	1	0.013848	0.01921056	0.721	0.4711
T3	1	0.033036	0.01932549	1.709	0.0876
T4	1	0.010153	0.01952022	0.520	0.6031
T5	1	0.014779	0.01975319	0.748	0.4545
T6	**1**	**0.032167**	**0.01980515**	**1.624**	**0.1046**
T7	**1**	**0.041914**	**0.01987062**	**2.109**	**0.0351**